Reconnection-mediated turbulence

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Waves and Complexity Thematic School — 20 May 2022, Porquerolles

Outline

1. **Introduction**

   ☞ Context & Motivation: turbulence in space and astrophysical plasmas

   ☞ Magnetic reconnection: basic concepts

   ☞ Phenomenology of Alfvénic turbulence: from weak to strong

2. **Further Developments in Theoretical Models**

   ☞ From dynamic alignment to reconnection-mediated regime

3. **Recent Progress via Numerical Simulations**

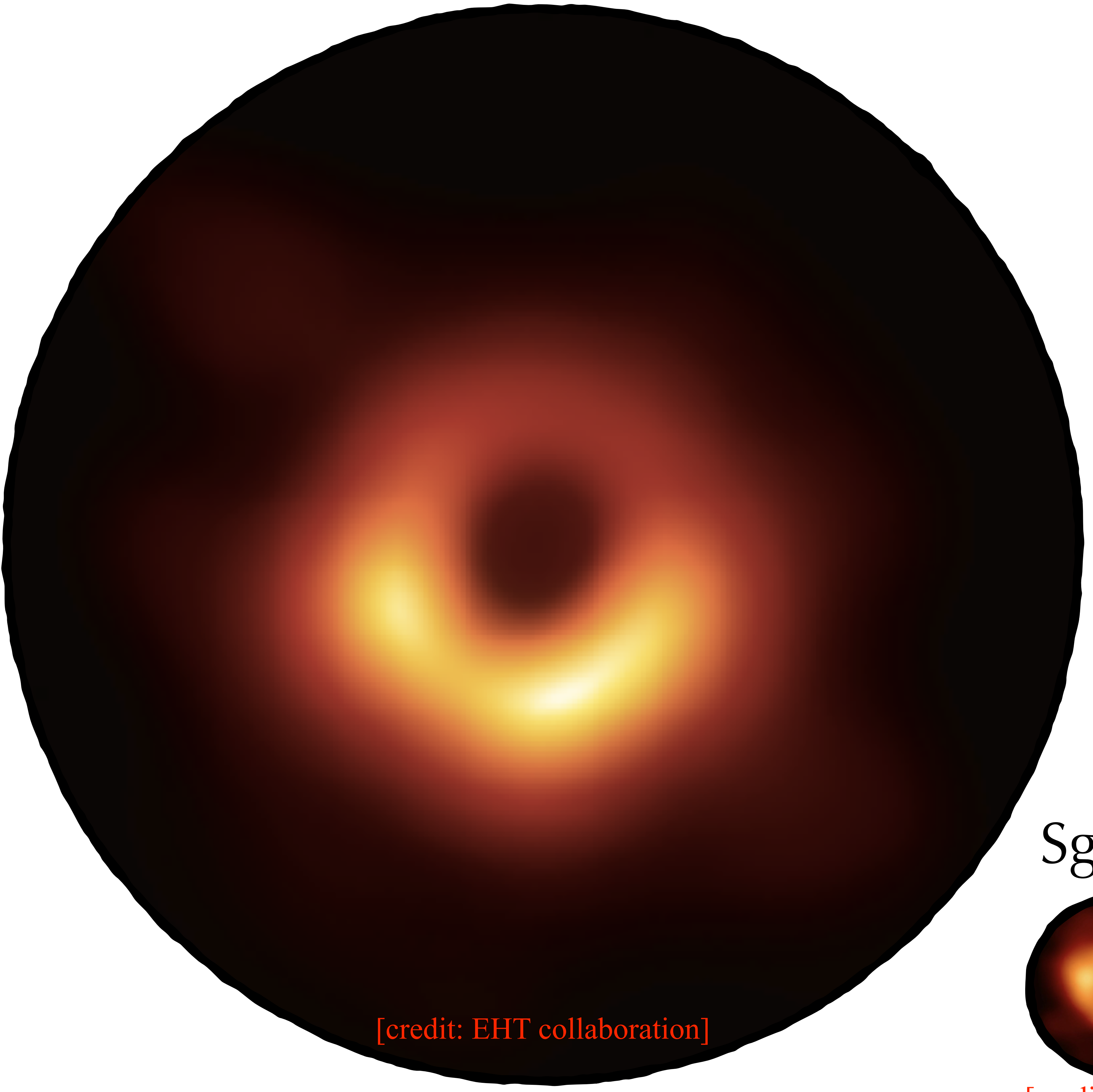
   ☞ 2D simulation of freely decaying magneto-hydrodynamic (MHD) turbulence

   ☞ 3D collisions of Alfvén-wave packets in reduced MHD

Context & Motivation

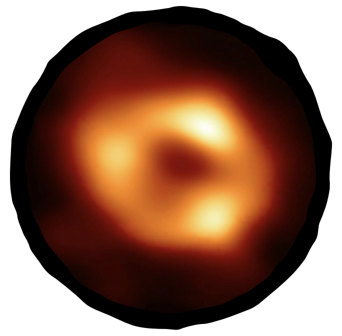
Turbulence in space and astrophysical plasmas

M87*



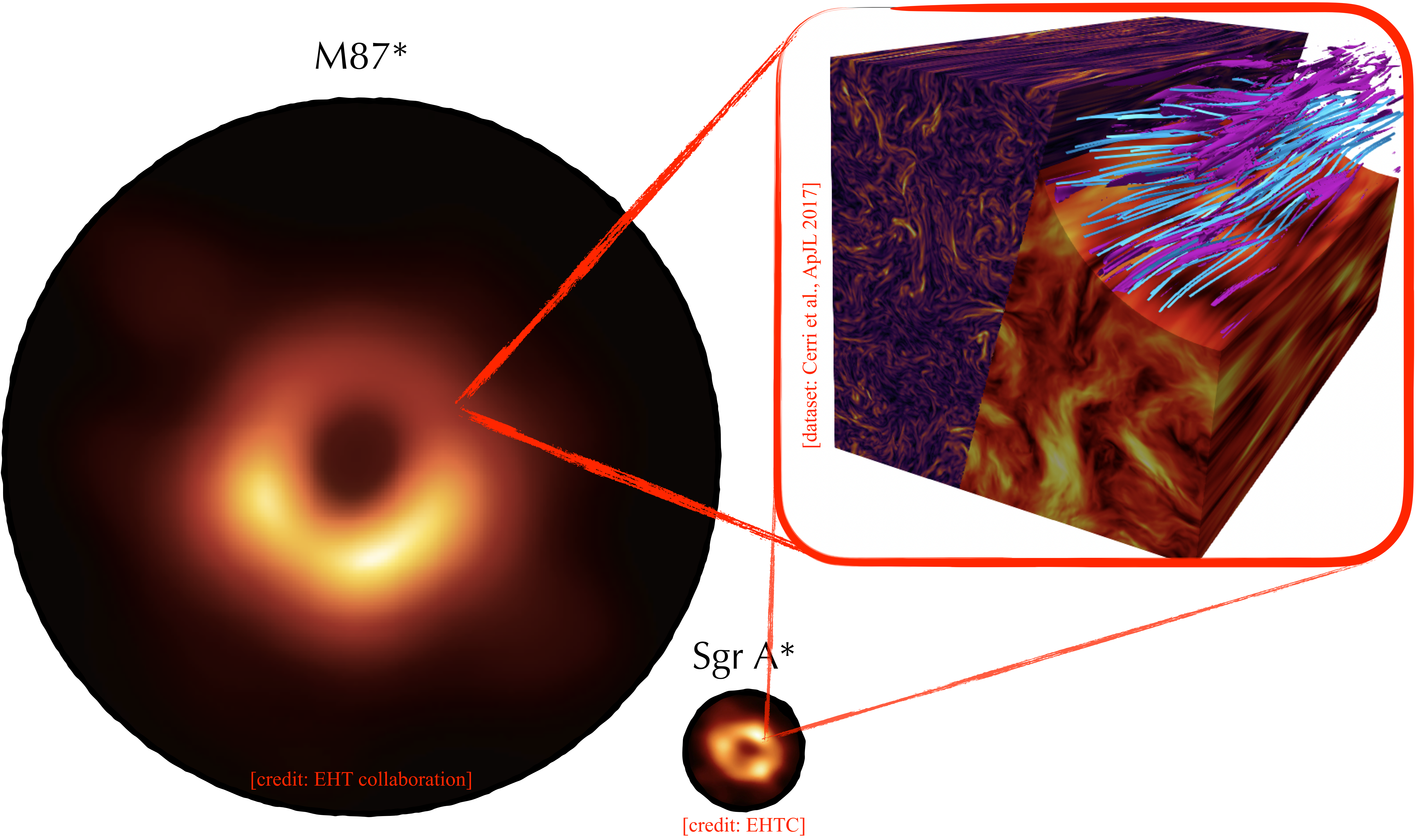
[credit: EHT collaboration]

Sgr A*



[credit: EHTC]

Turbulence in space and astrophysical plasmas



Turbulence in space and astrophysical plasmas

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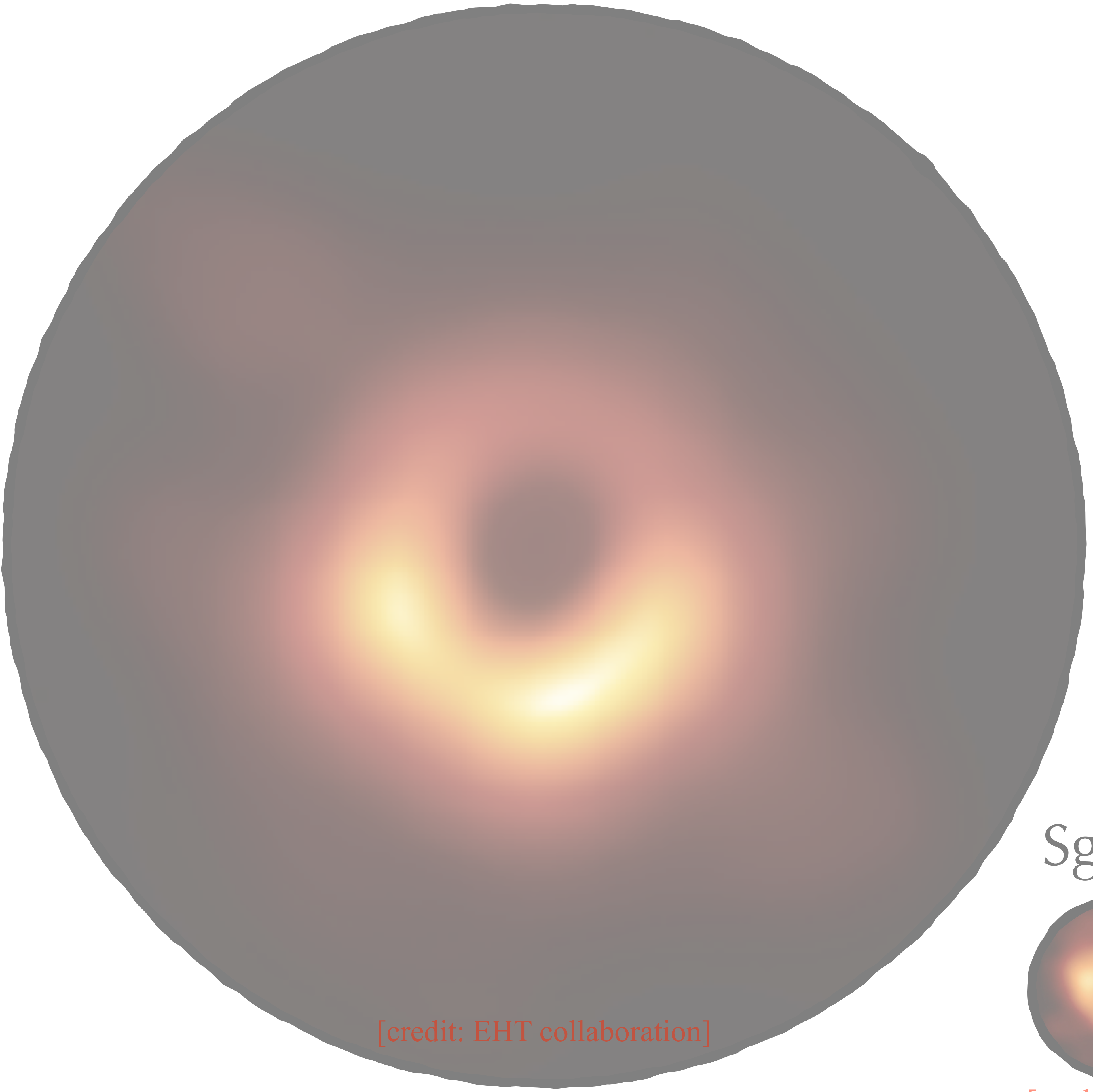

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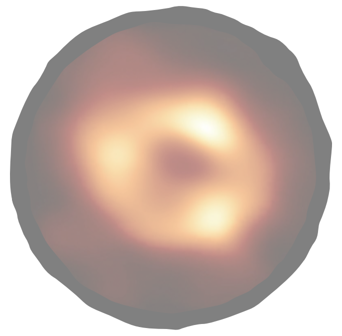

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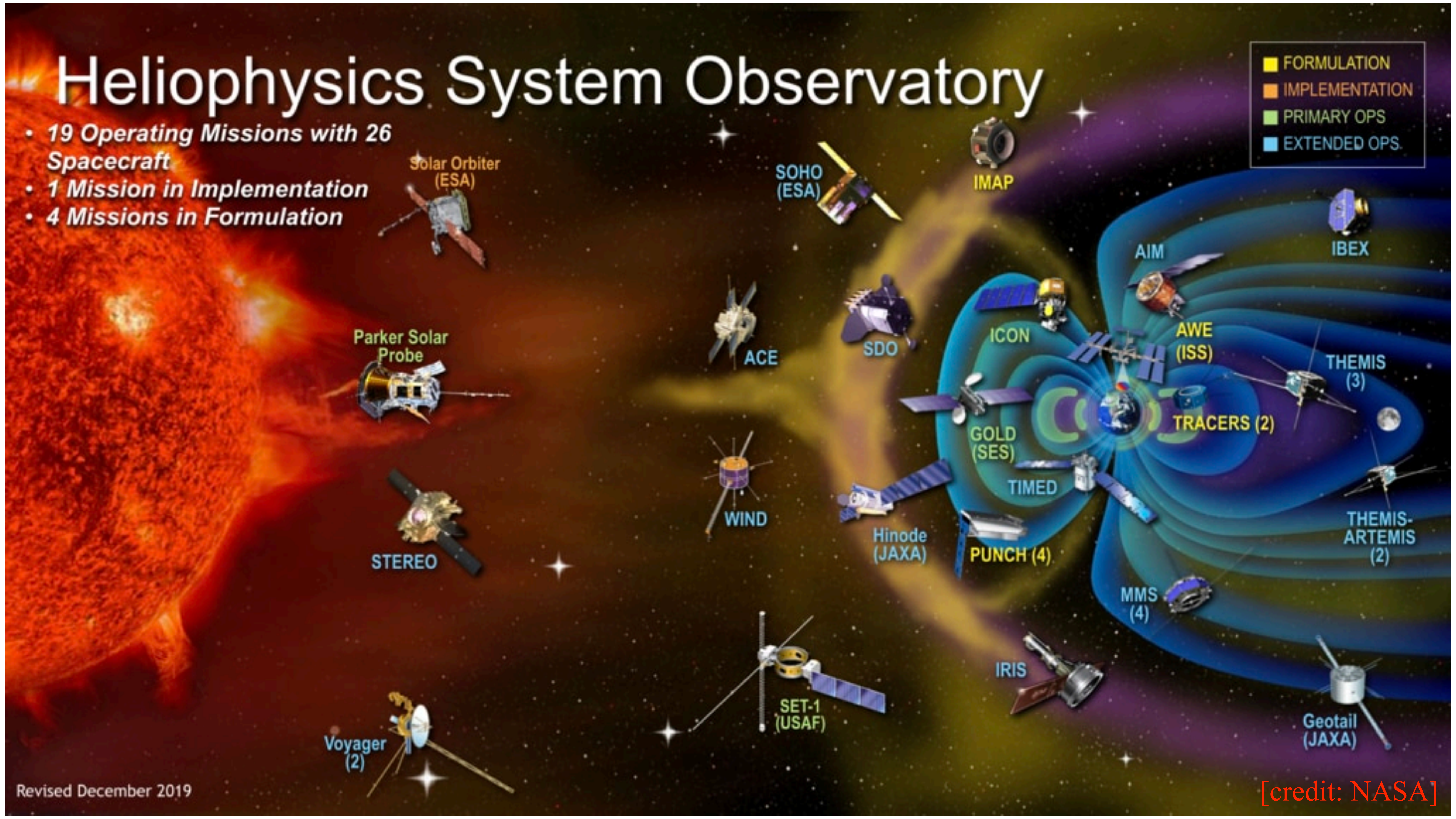
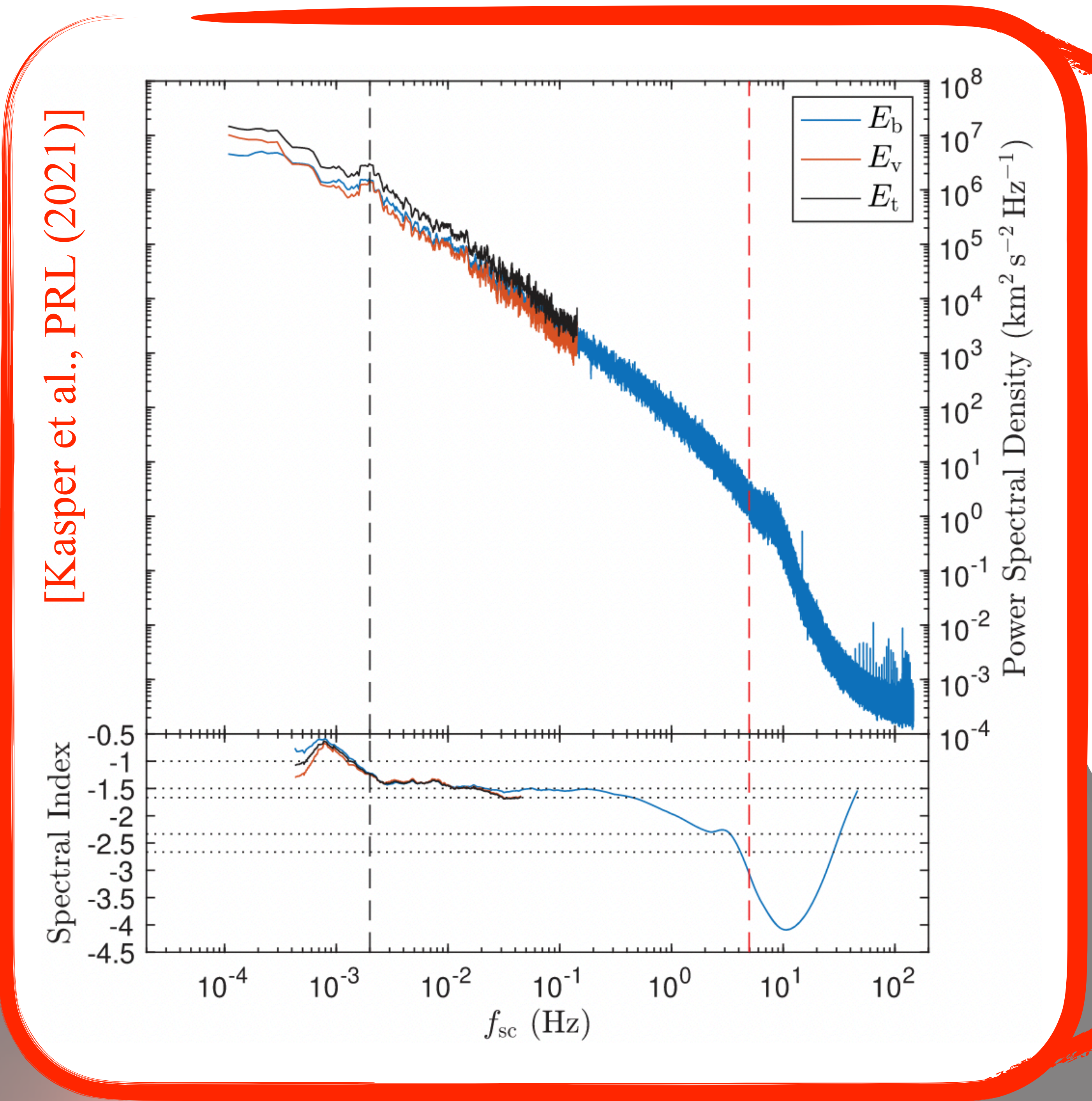
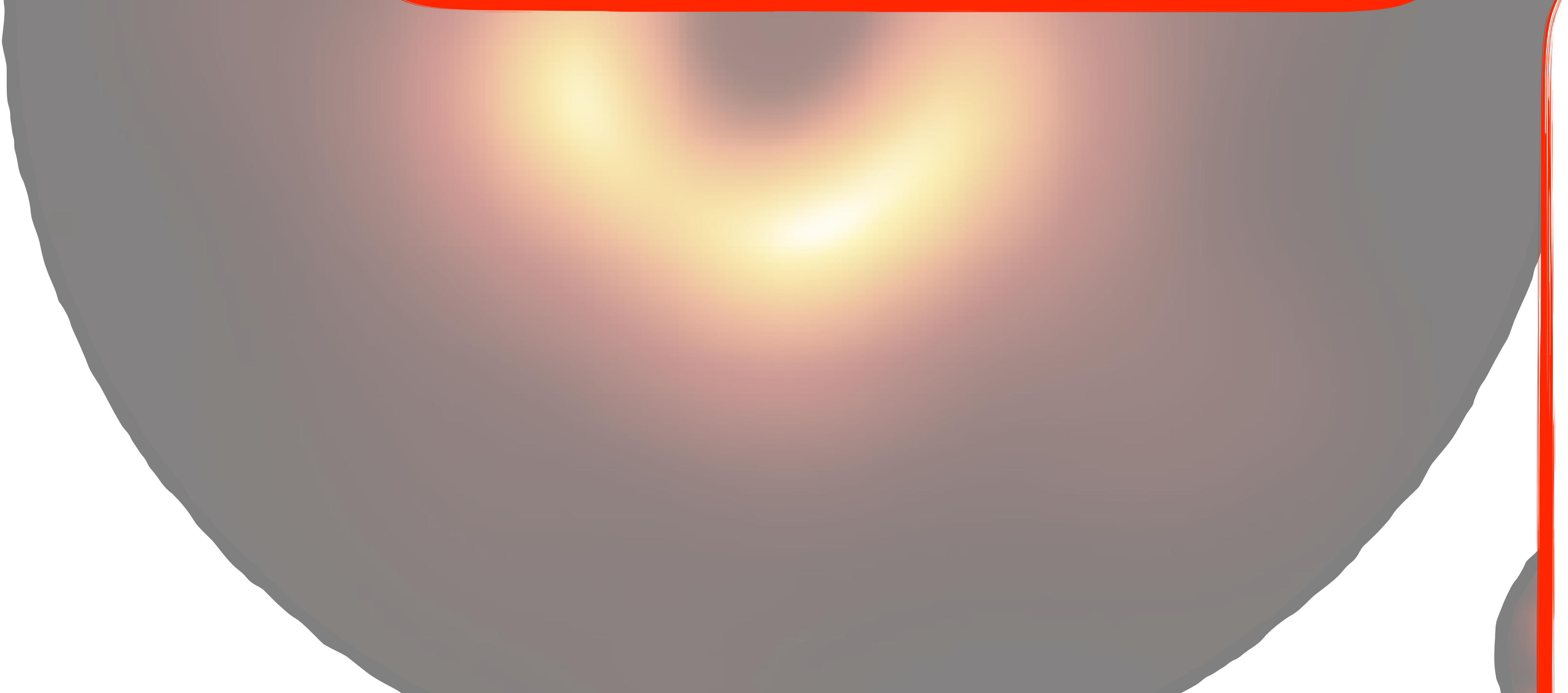


[credit: NASA]

Turbulence in space and astrophysical plasmas



[Kasper et al., PRL (2021)]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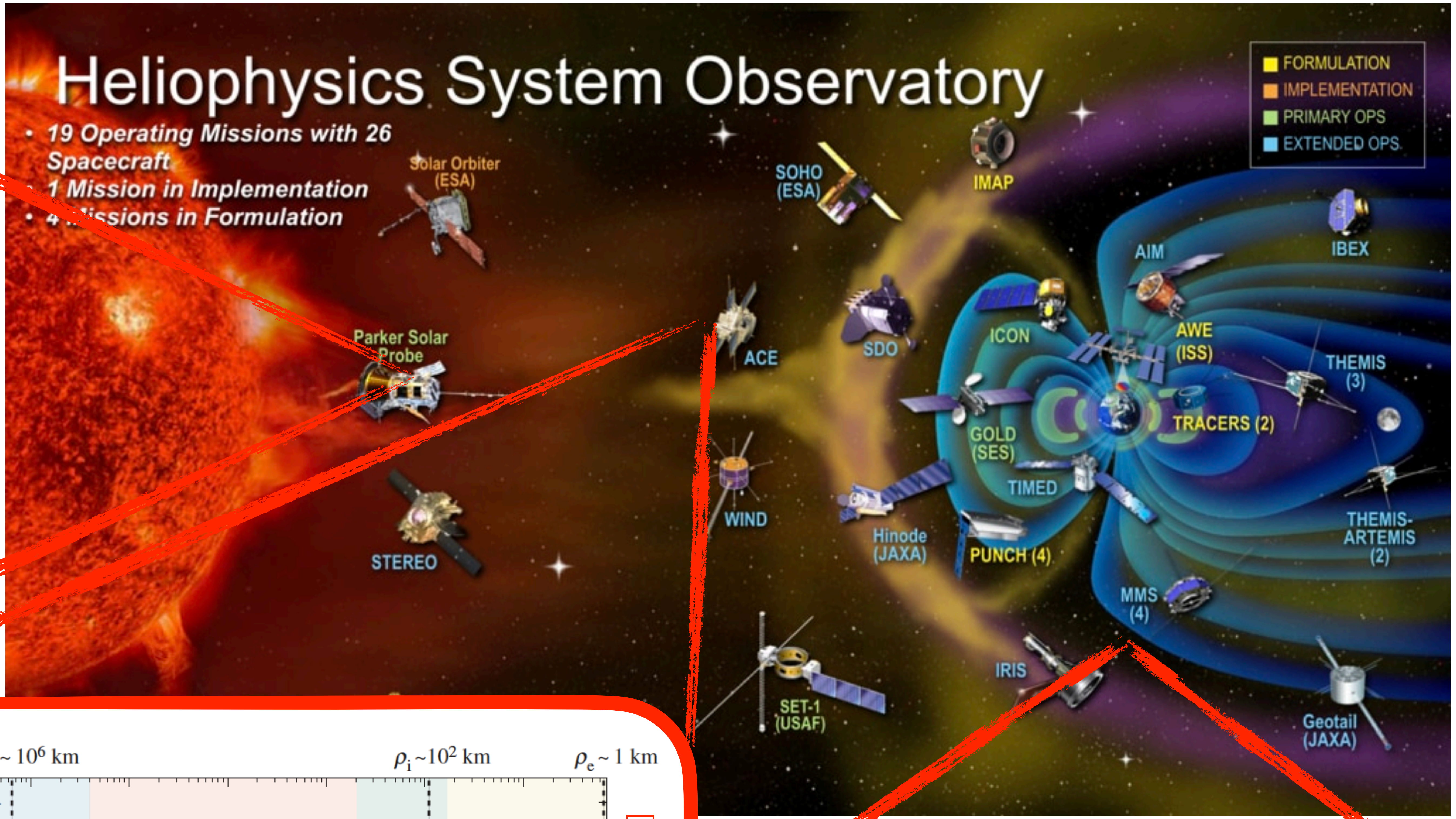
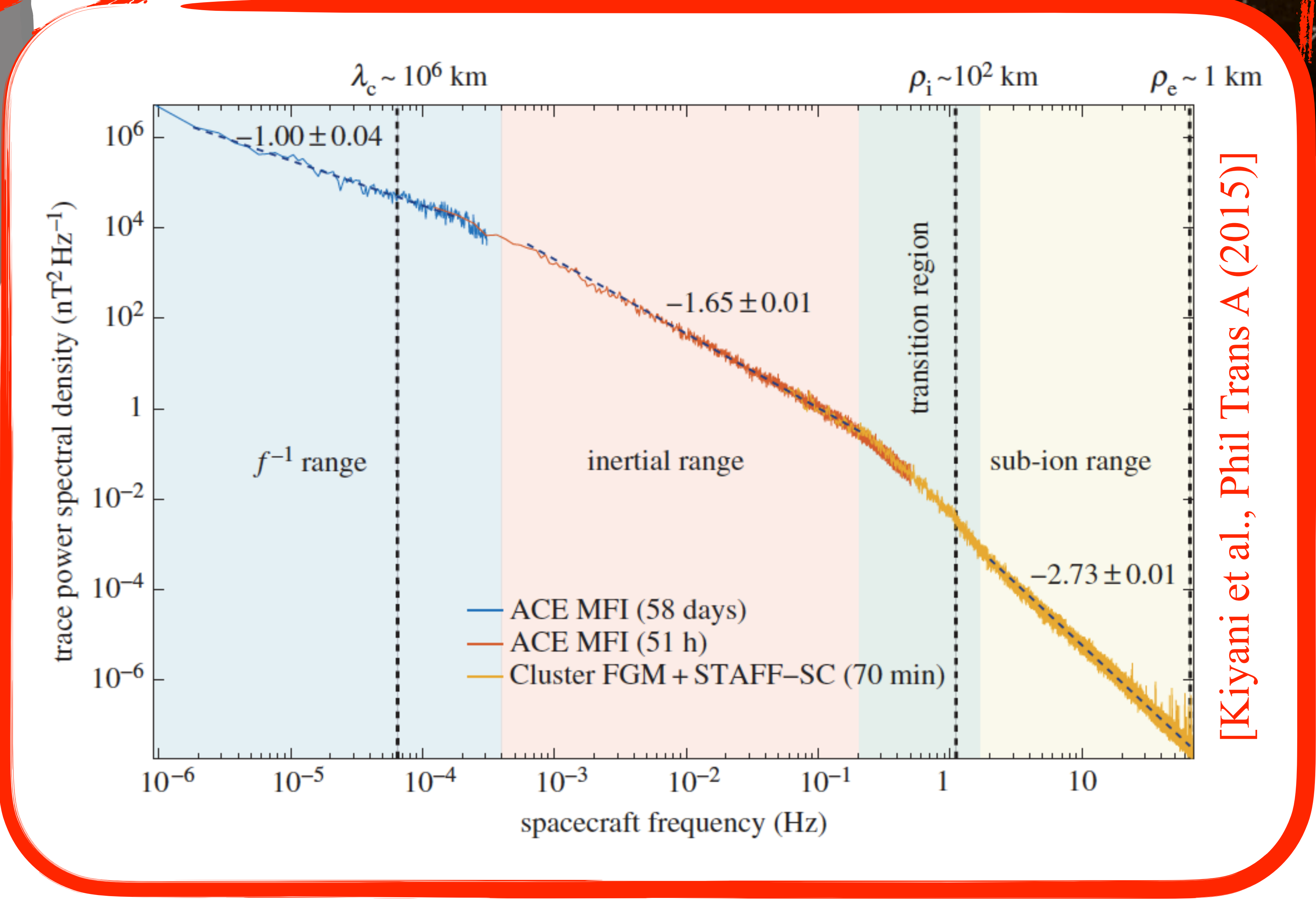
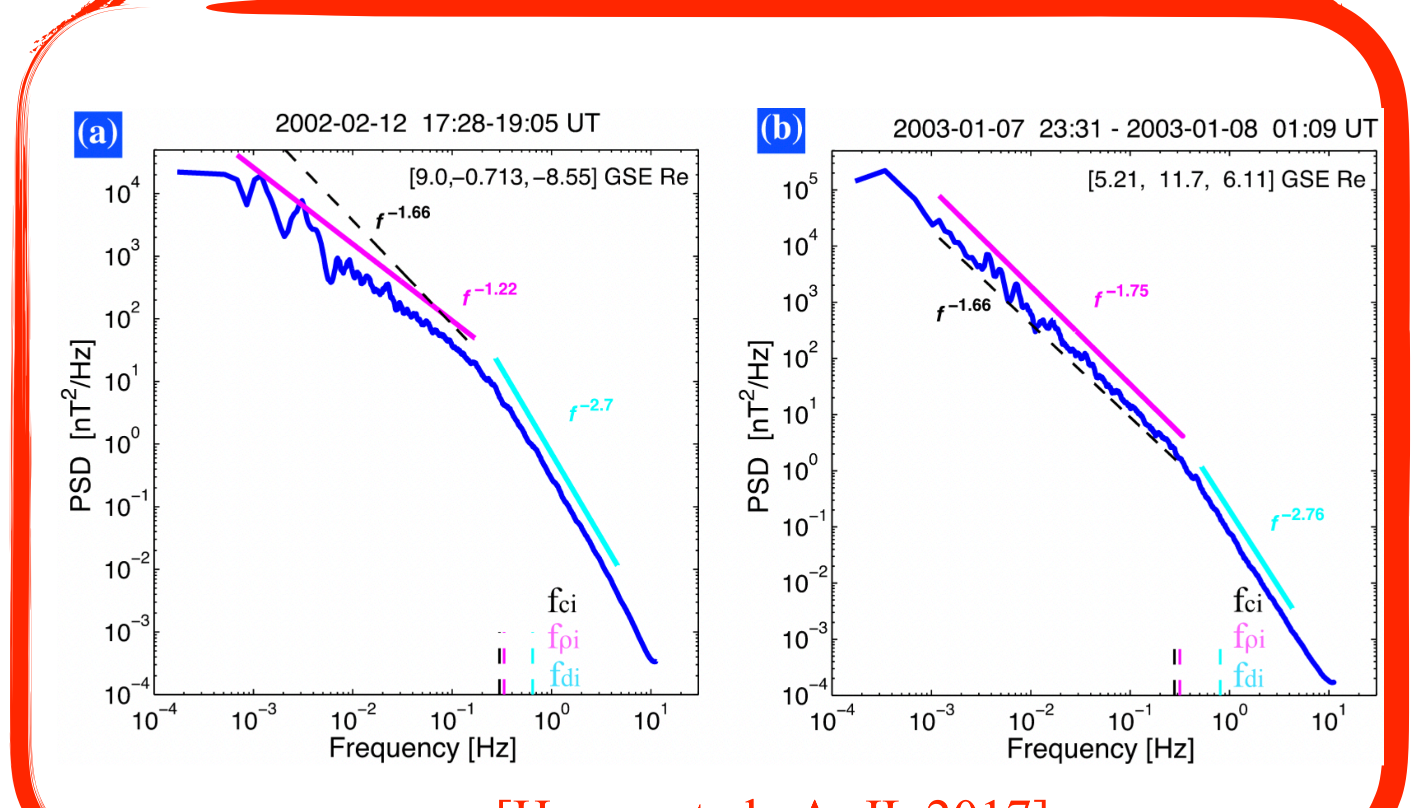


[credit: NASA]



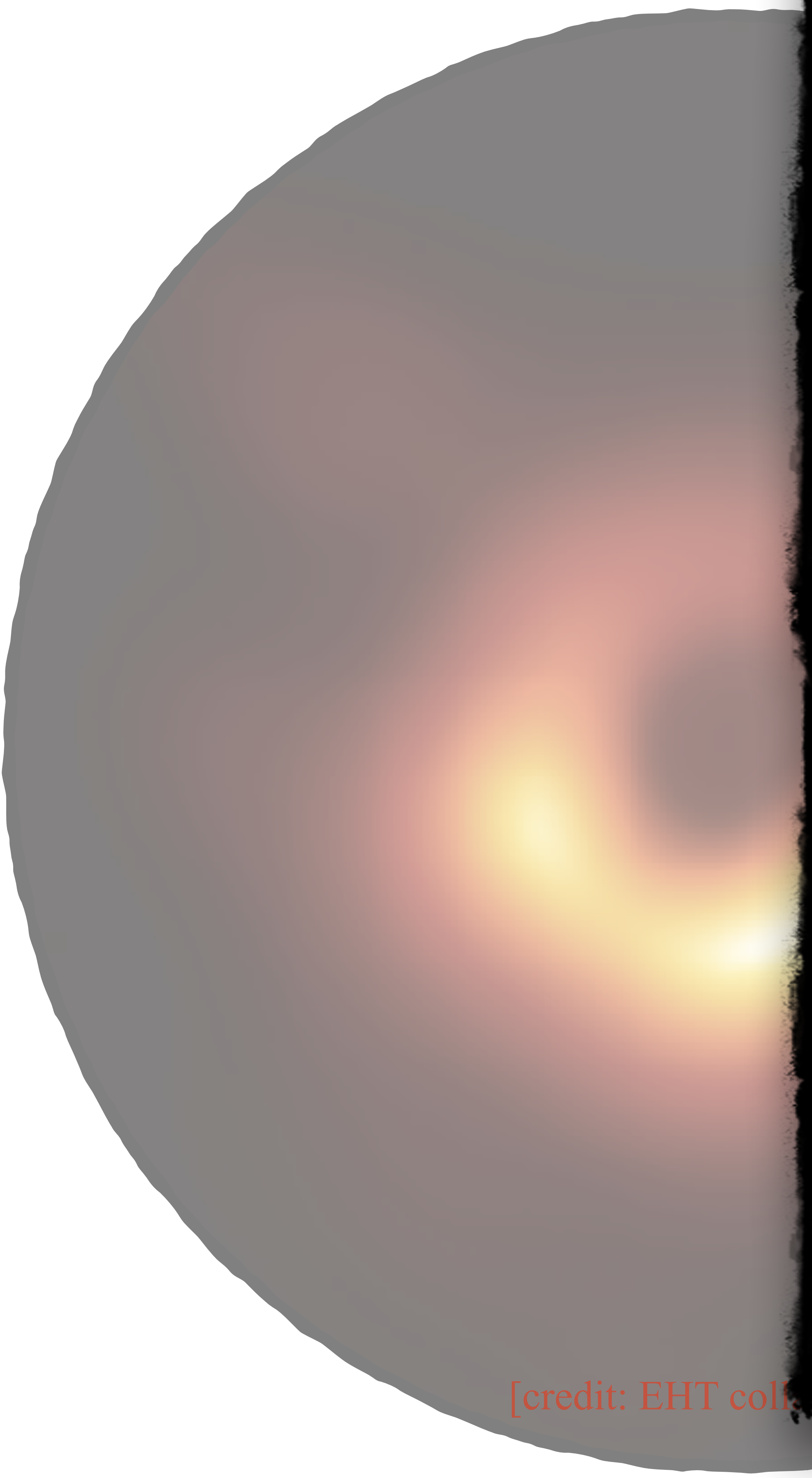
[Kiyani et al., Phil Trans A (2015)]



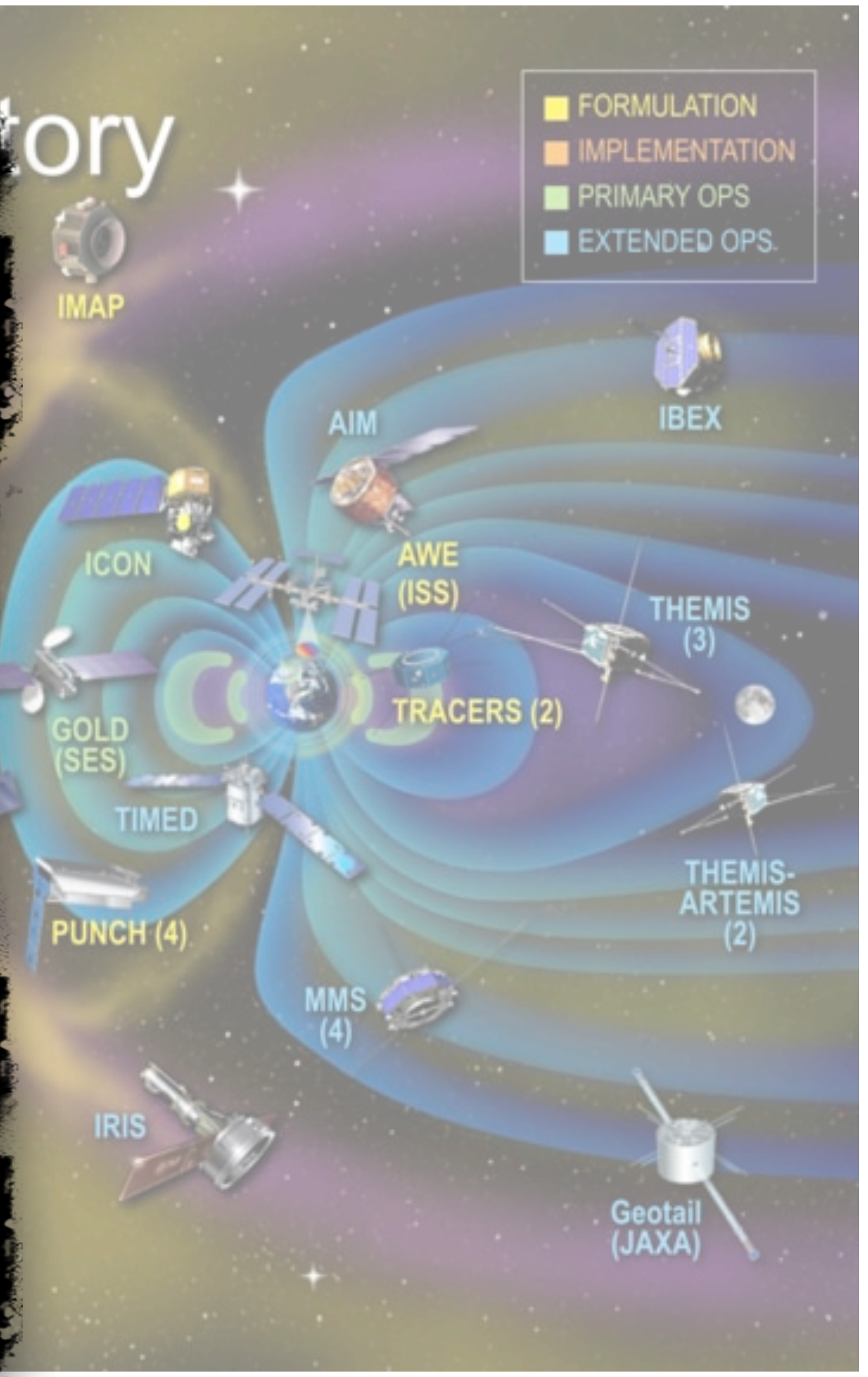
[Huang et al., ApJL 2017]

Turbulence in space and astrophysical plasmas

M87



[credit: EHT col...]



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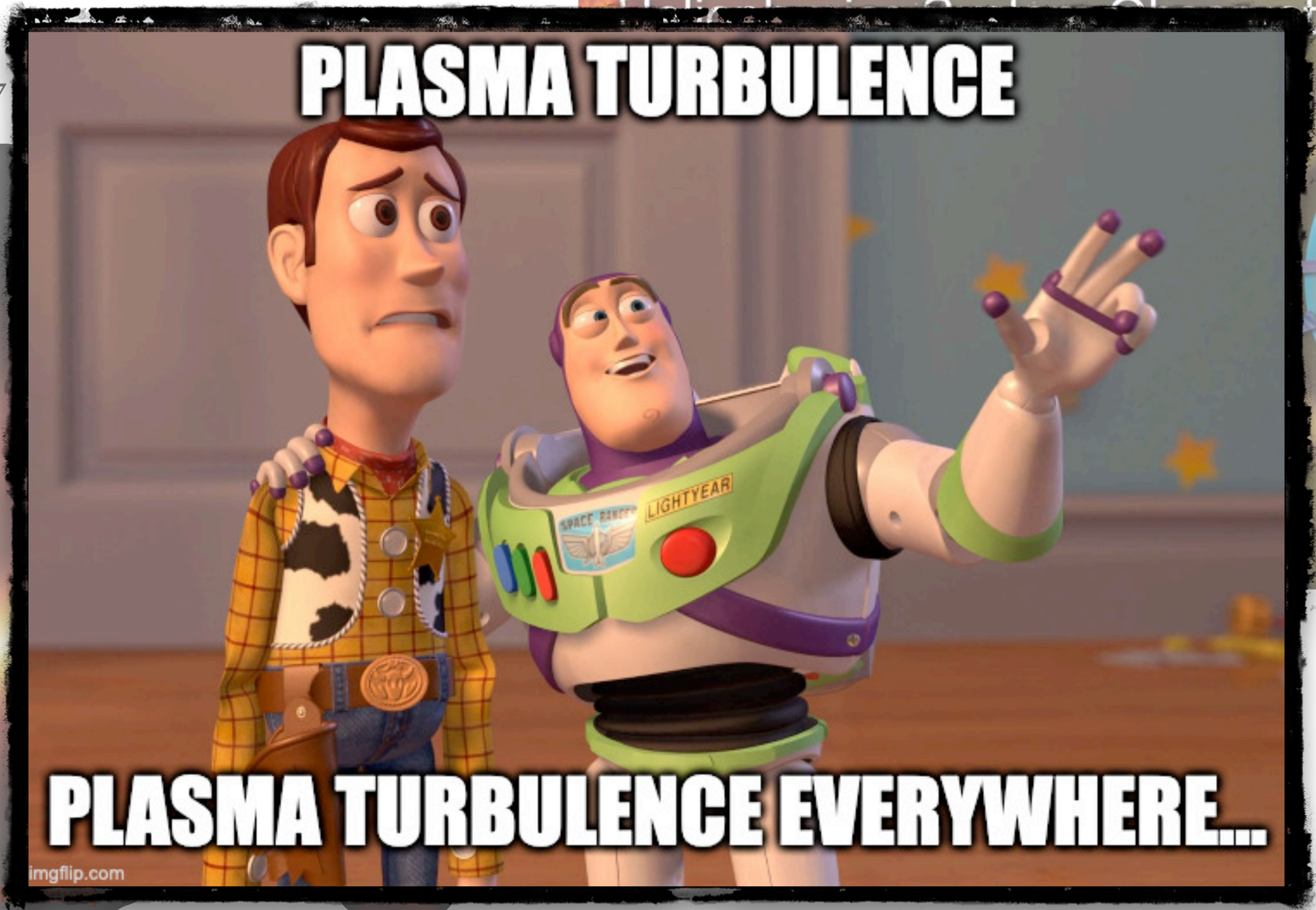

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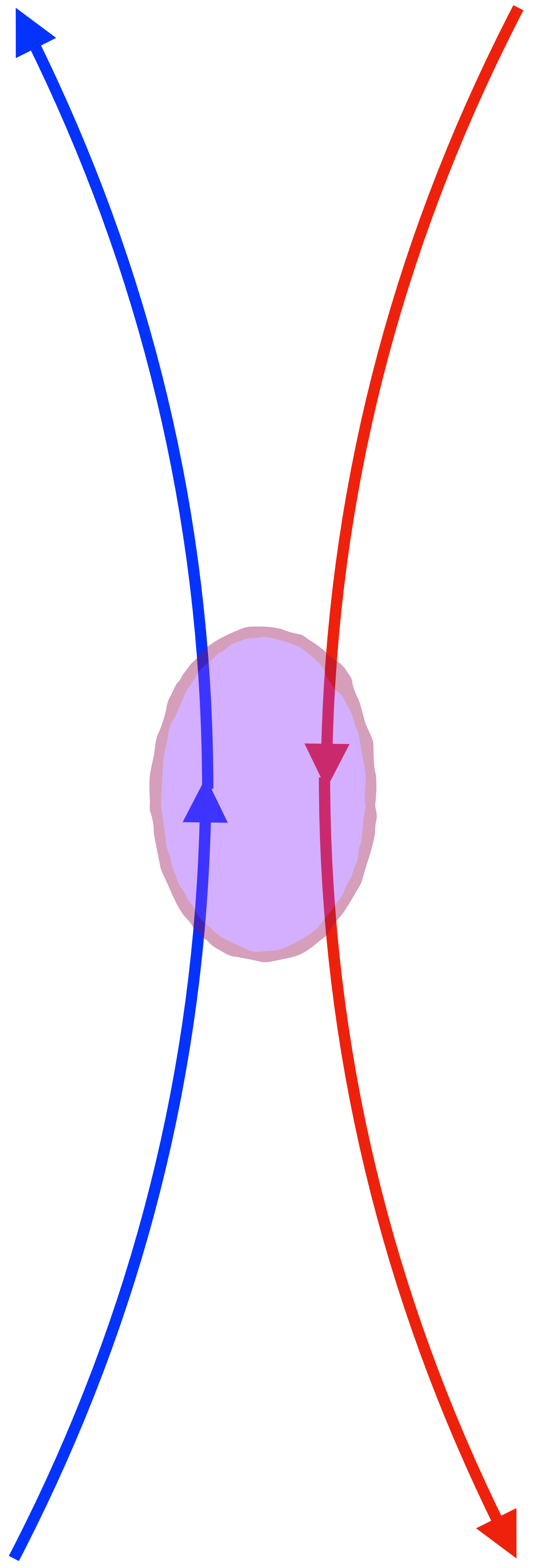
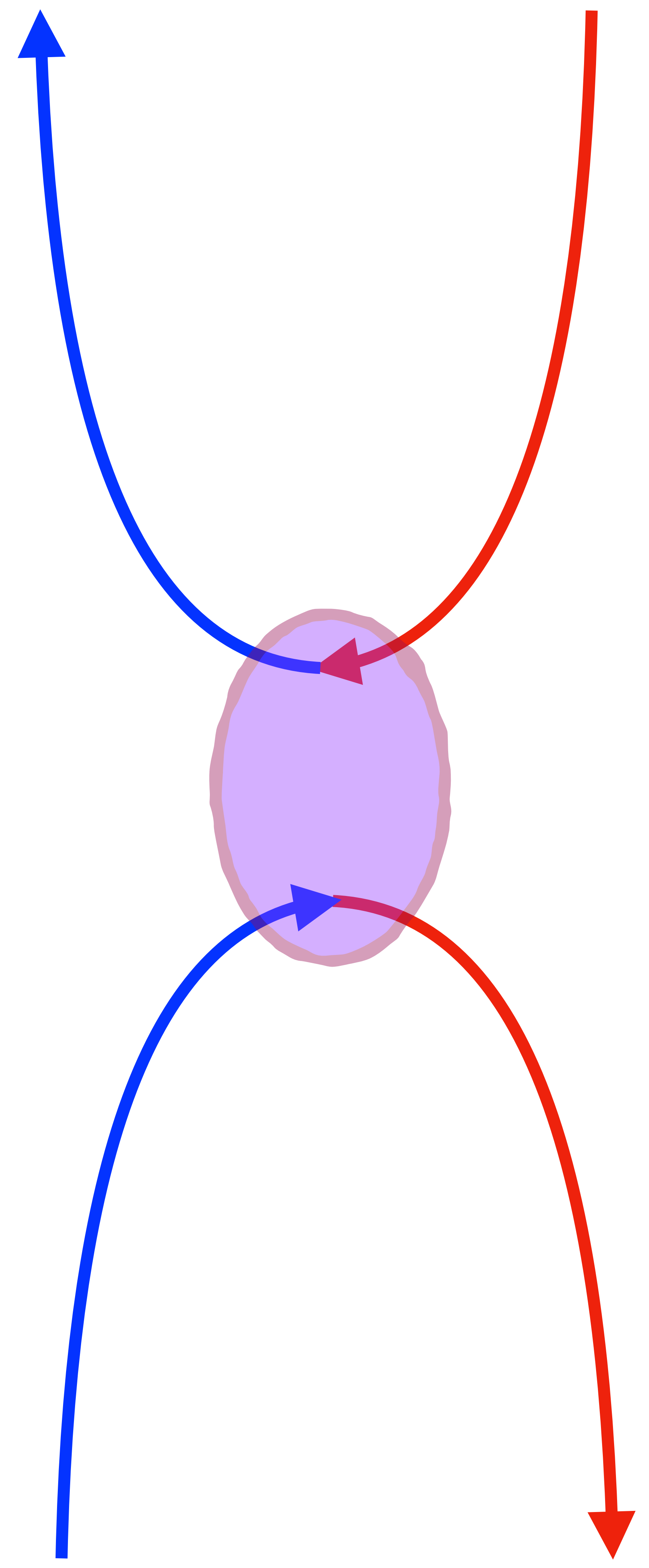
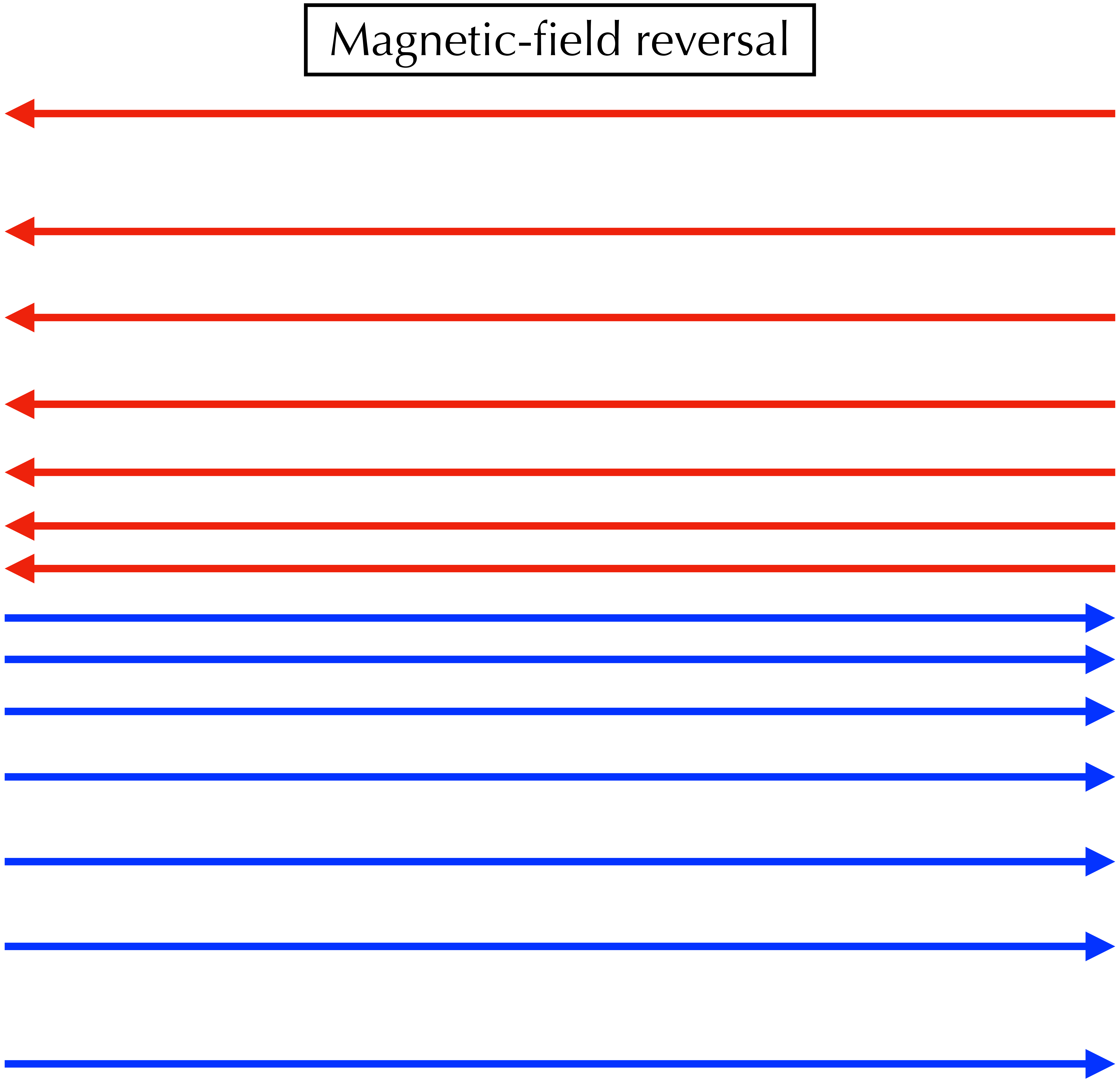


Magnetic Reconnection



Magnetic reconnection: basic concepts



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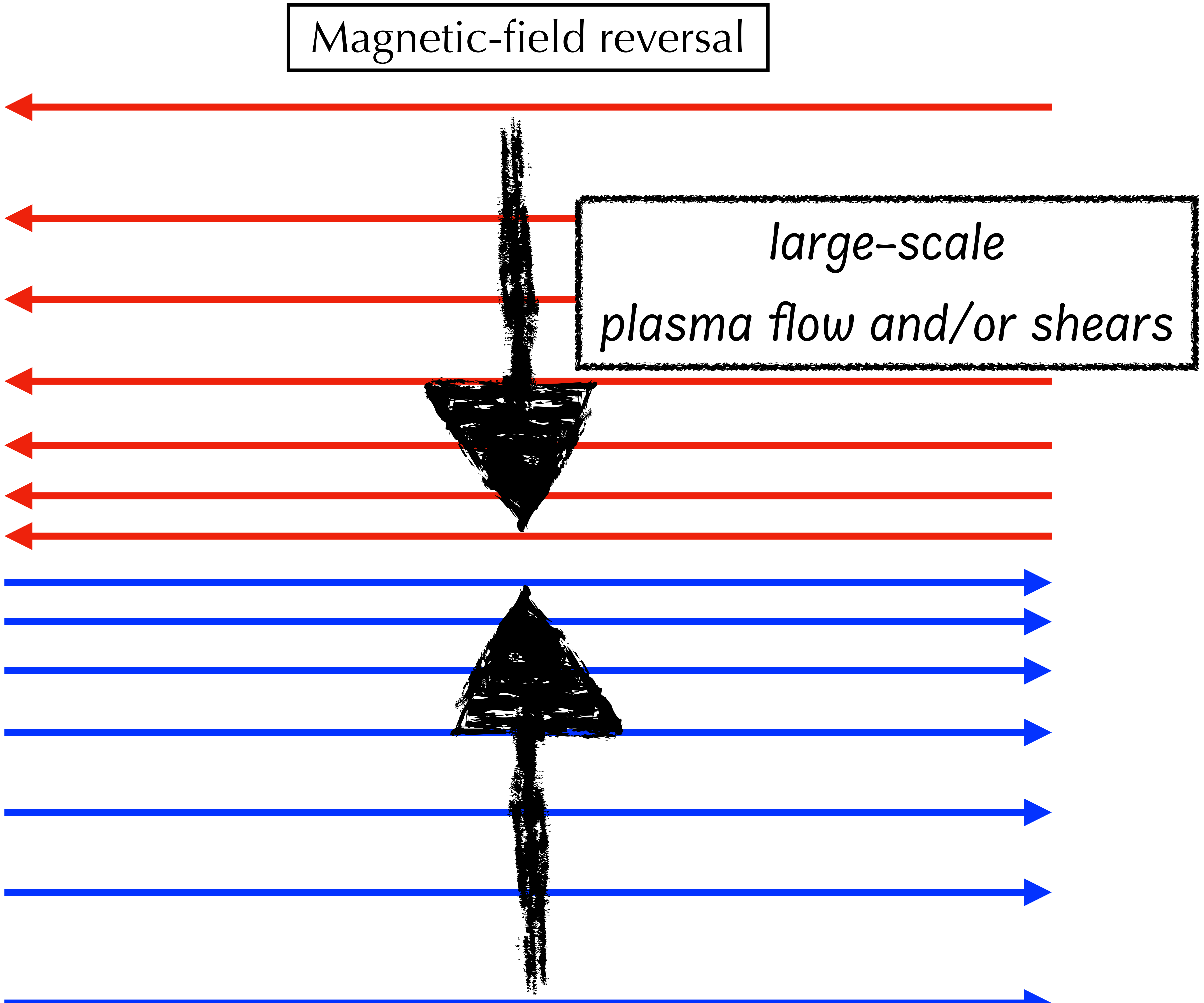

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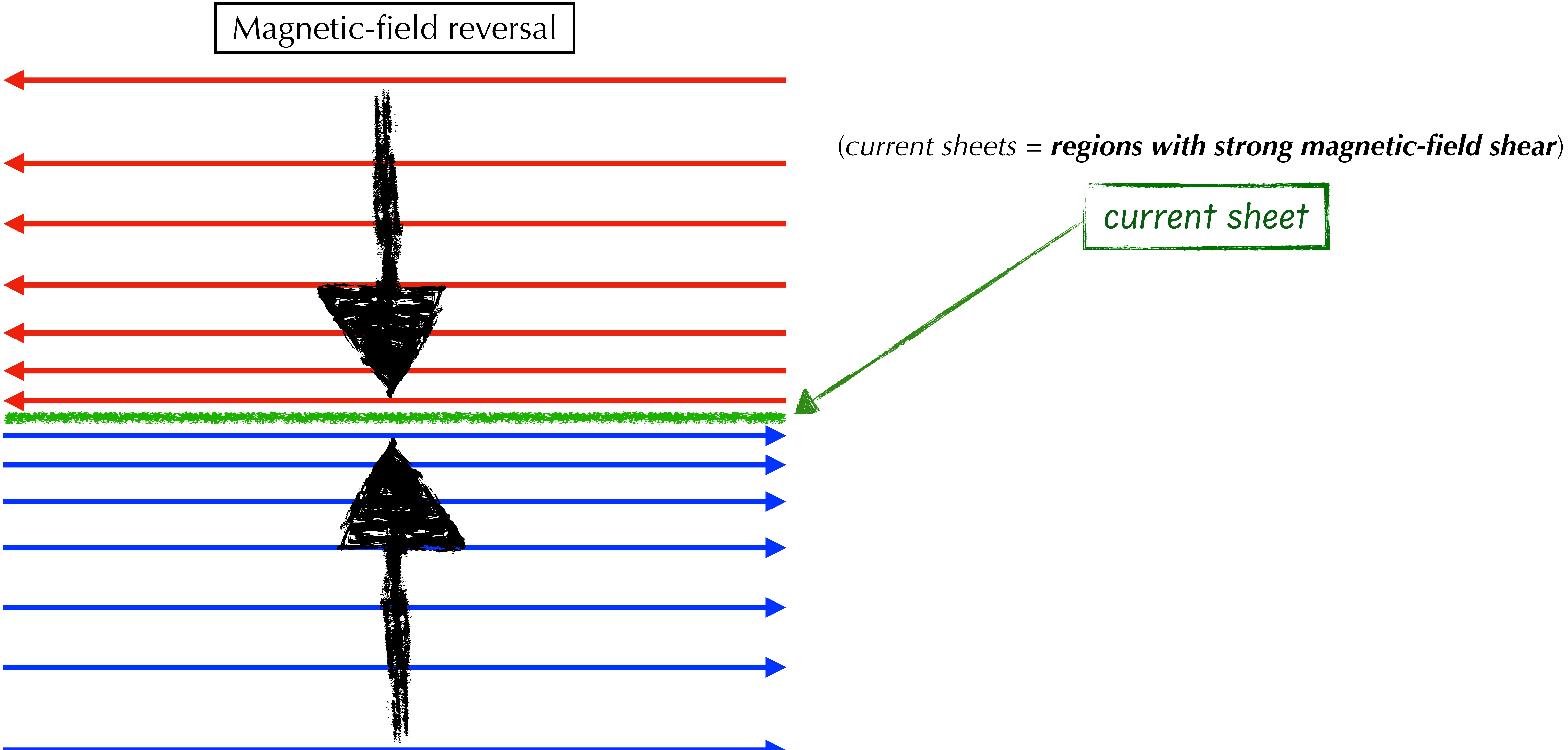


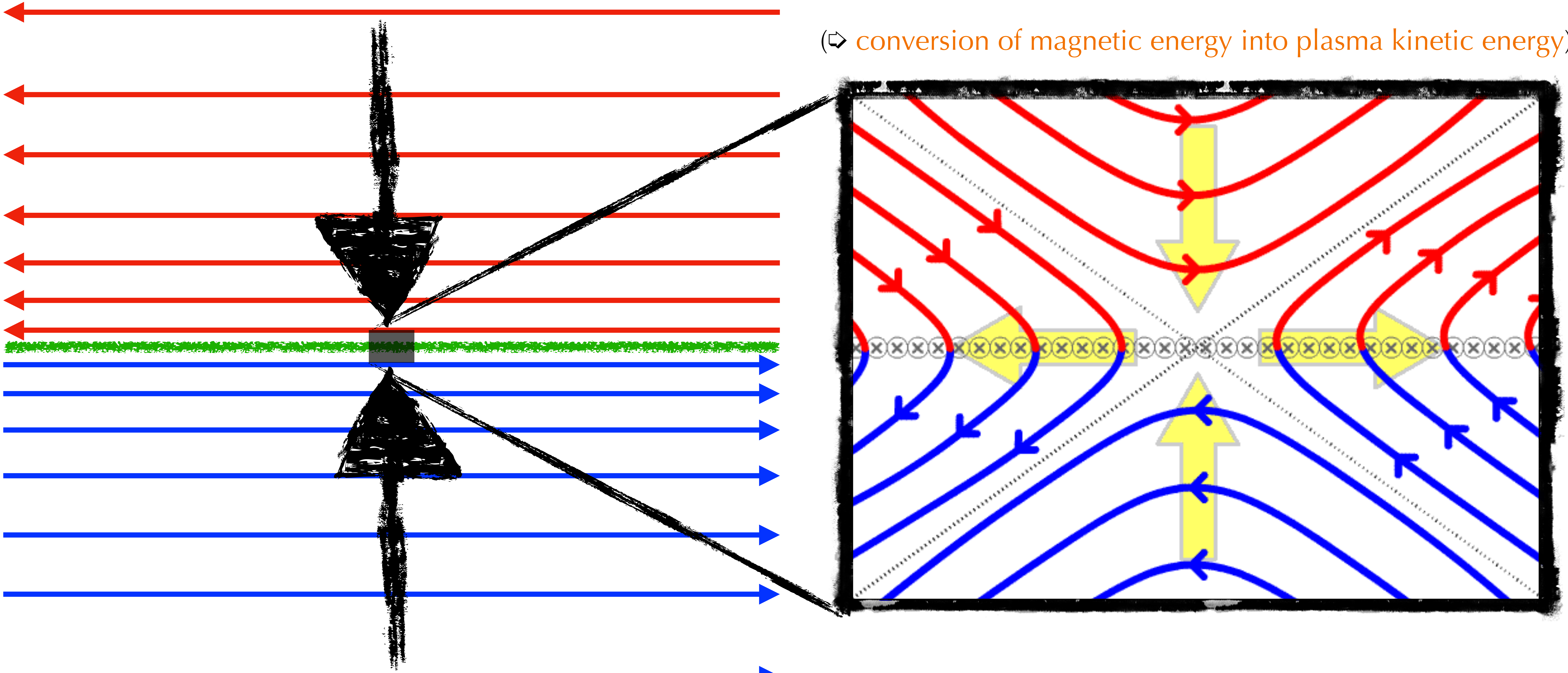
Magnetic reconnection: basic concepts

Magnetic-field reversal

if the magnetic shear is ***“strong enough***(*)***”***

⇒ **RECONNECTION!**

(➪ conversion of magnetic energy into plasma kinetic energy)



[gif from Wikipedia]

(*) there is a parameter called Δ′ … but this would require an entire lecture!

☞ ask Camille Granier and Emanuele Tassi, they know everything about it!

Magnetic reconnection: basic concepts

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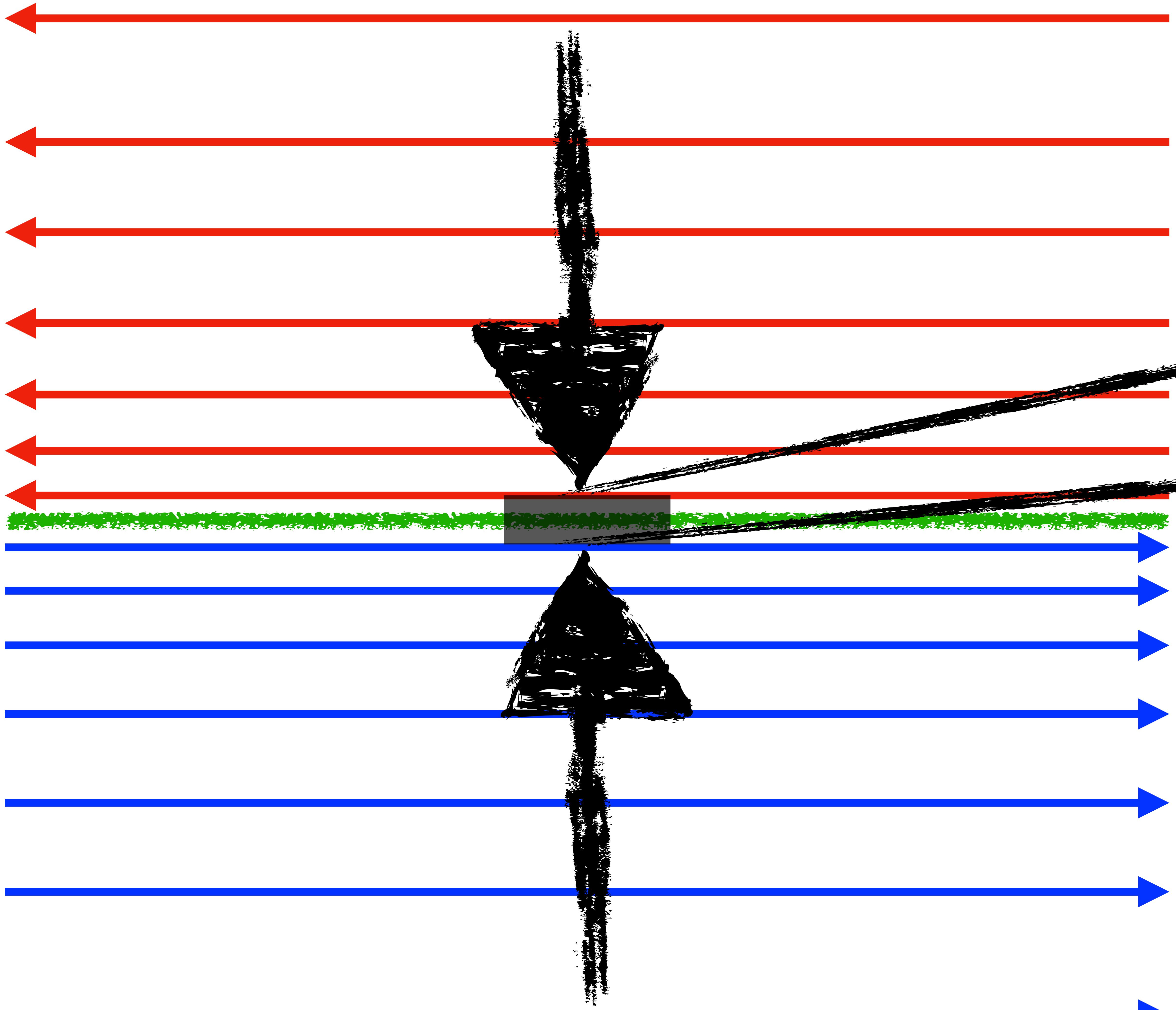

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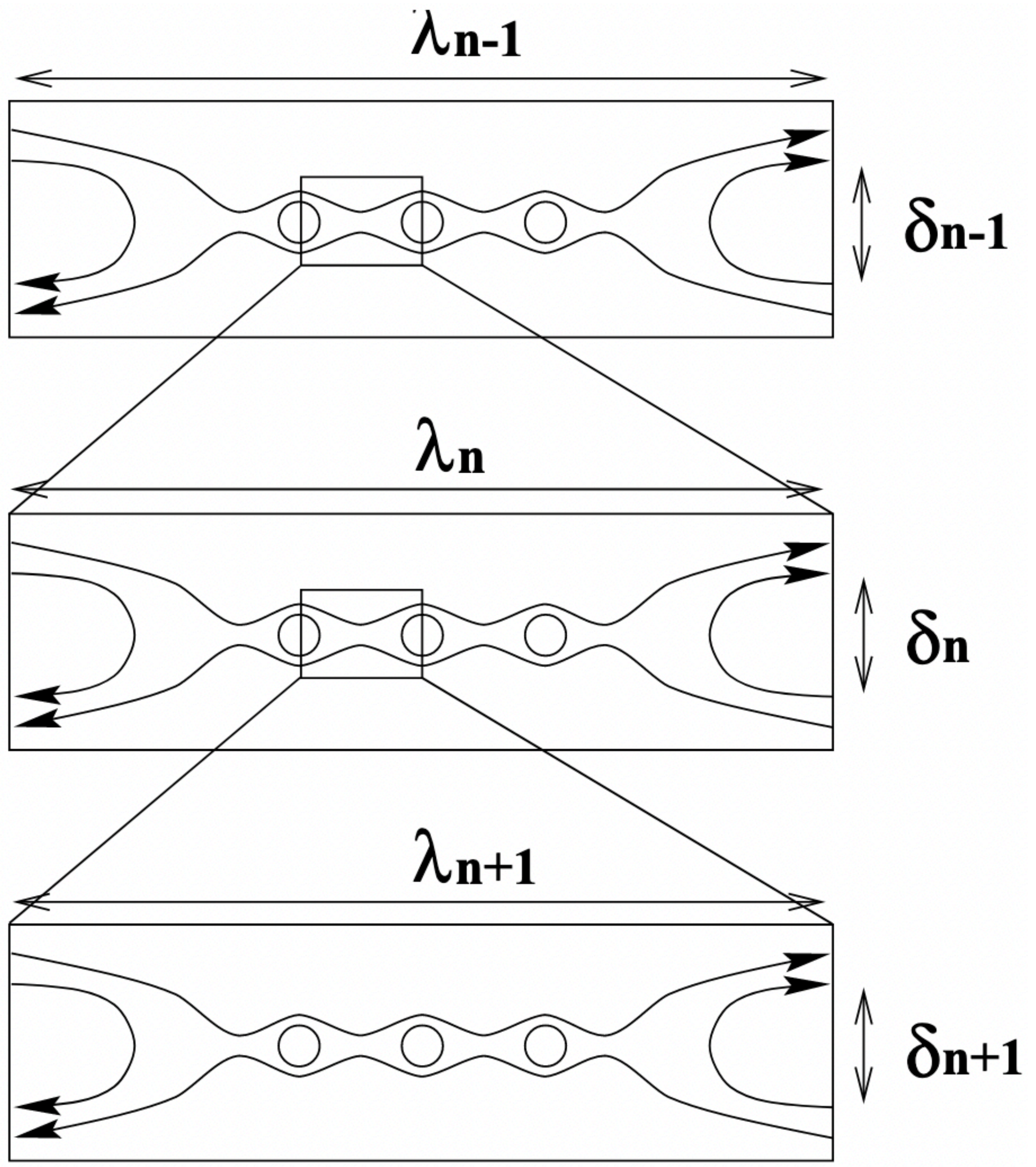
“PLASMOID” regime with recursive/fractal reconnection



[Shibata & Tanuma, EPS 2001]

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Magnetic reconnection: basic concepts

☞ magnetic reconnection requires to **break magnetic-flux conservation**:

this is done *only by **non-ideal MHD effects*** (*non-negligible at "small" scales!*)

1. generalized Ohm's law: $\boldsymbol{E} + \dfrac{\boldsymbol{u}_{\mathrm{i}} \times \boldsymbol{B}}{c} = \boldsymbol{R}$

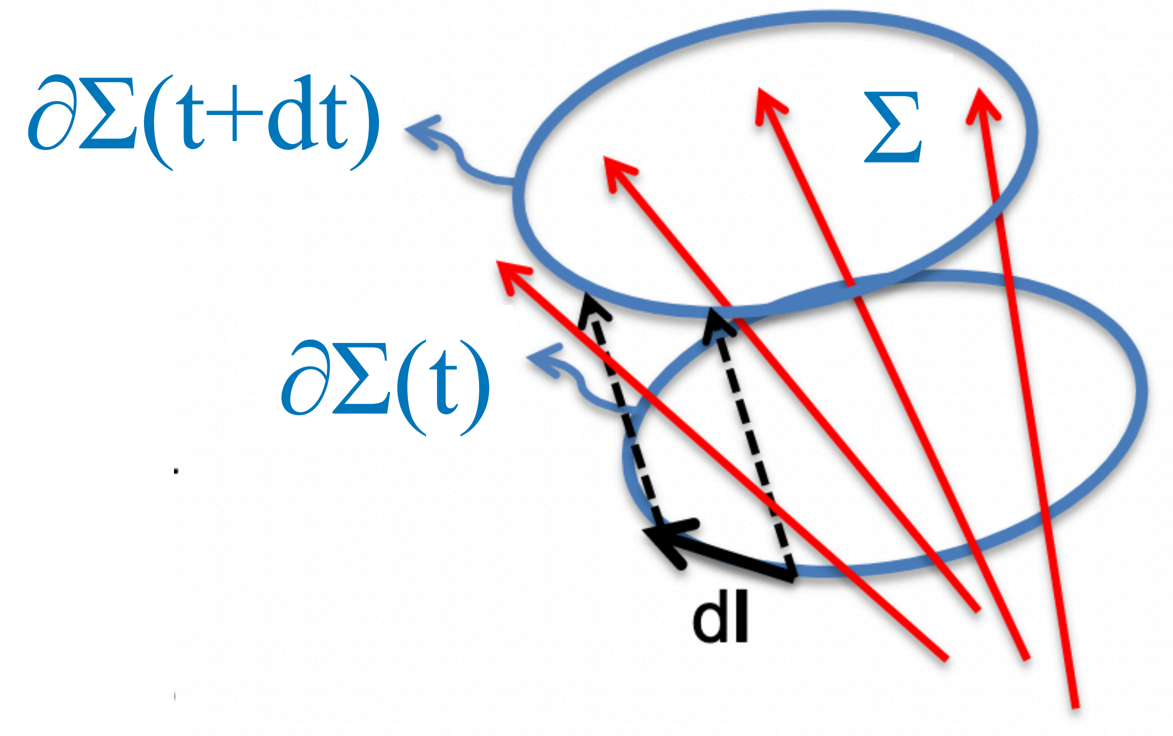
R = 0 in ideal MHD

$$\boldsymbol{R} \equiv \underbrace{\eta \boldsymbol{J}}_{\text{(I)}} + \underbrace{\frac{\boldsymbol{J} \times \boldsymbol{B}}{nec}}_{\text{(II)}} - \underbrace{\frac{\boldsymbol{\nabla} \cdot \overline{\overline{\boldsymbol{P}}}_{\mathrm{e}}}{ne}}_{\text{(III)}} + \underbrace{\frac{m_{\mathrm{e}}}{ne^2}\left[\boldsymbol{\nabla} \cdot \left(\boldsymbol{J}\boldsymbol{u}_{\mathrm{i}} + \boldsymbol{u}_{\mathrm{i}}\boldsymbol{J} - \frac{\boldsymbol{J}\boldsymbol{J}}{ne}\right) + \frac{\partial \boldsymbol{J}}{\partial t}\right]}_{\text{(IV)}}$$

2. Faraday's law: $\dfrac{\partial \boldsymbol{B}}{\partial t} = -c\,\boldsymbol{\nabla} \times \boldsymbol{E}$

$$\Rightarrow \quad \frac{\mathrm{d}\Phi_{\boldsymbol{B}}[\Sigma]}{\mathrm{d}t} \equiv \frac{\mathrm{d}}{\mathrm{d}t}\int_\Sigma \boldsymbol{B} \cdot \mathbf{d}\boldsymbol{S} \equiv \int_\Sigma \partial_t \boldsymbol{B} \cdot \mathbf{d}\boldsymbol{S} + \oint_{\partial\Sigma} \boldsymbol{B} \cdot (\boldsymbol{u} \times \mathbf{d}\boldsymbol{\ell})$$

$$= -c \int_\Sigma (\boldsymbol{\nabla} \times \boldsymbol{R}) \cdot \mathbf{d}\boldsymbol{S}$$



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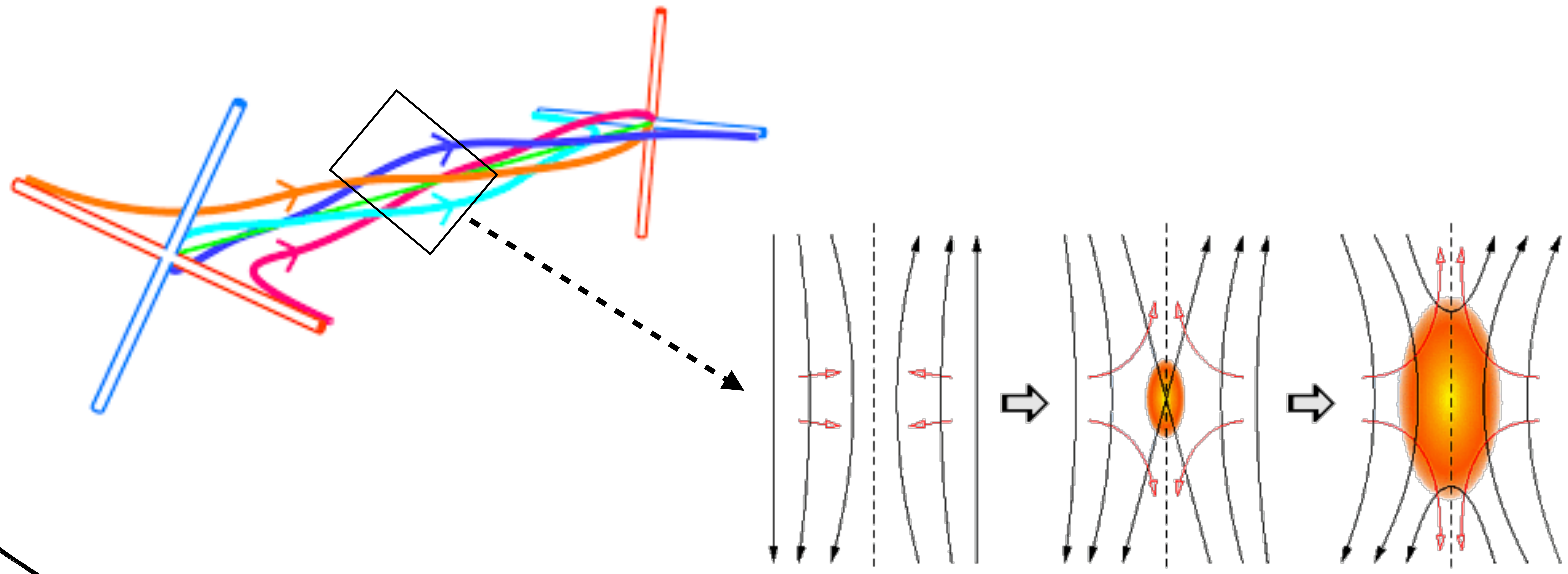
...or Emanuele Tassi, they know everything about it!

Magnetic reconnection: basic concepts

☞ **plasma turbulence:**

1. ***naturally develops current sheets***
2. ***naturally generates small scales*** *(eventually "activating" non-ideal MHD effects)*

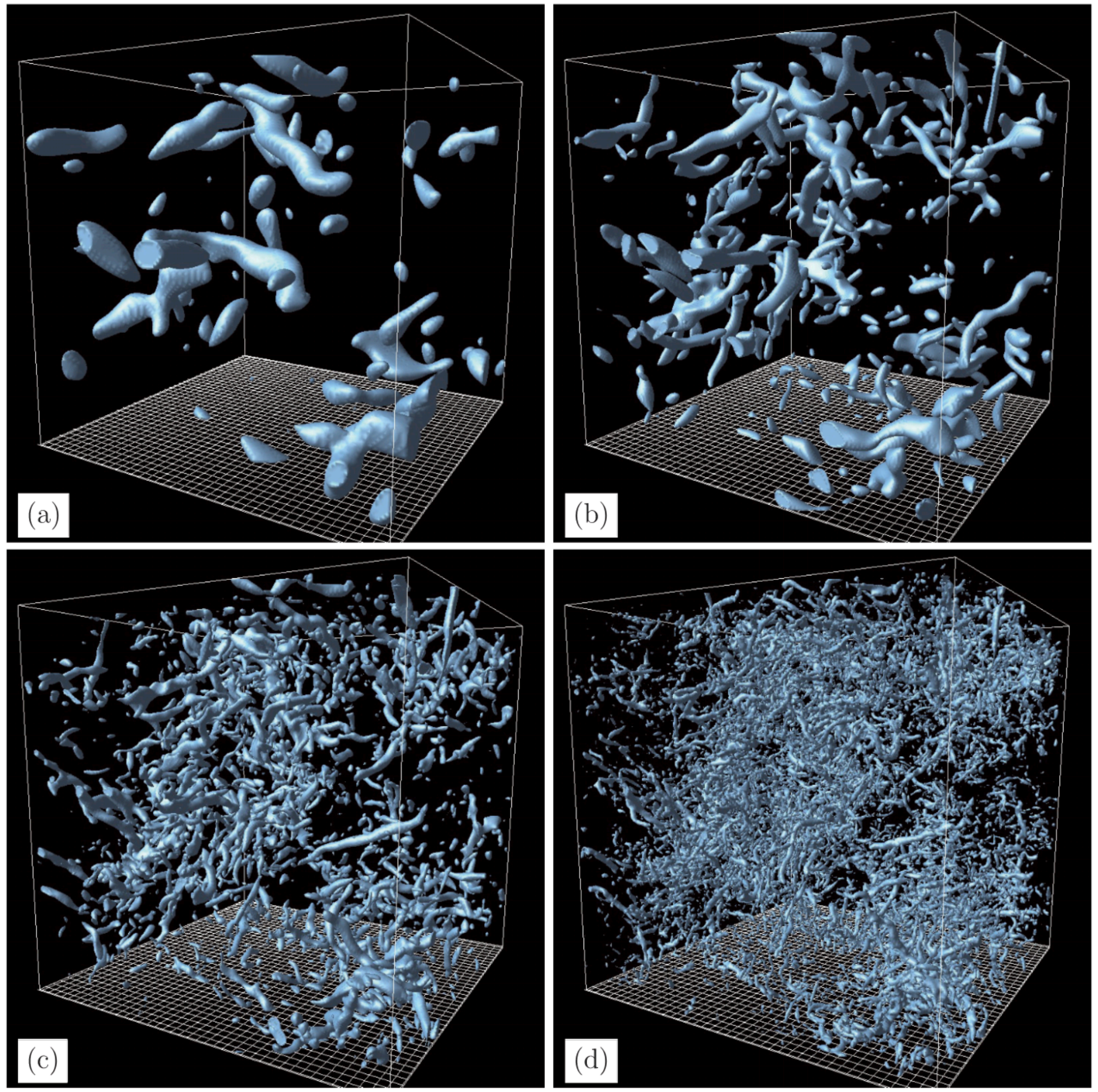



Alfvénic Turbulence

Alfvénic Turbulence

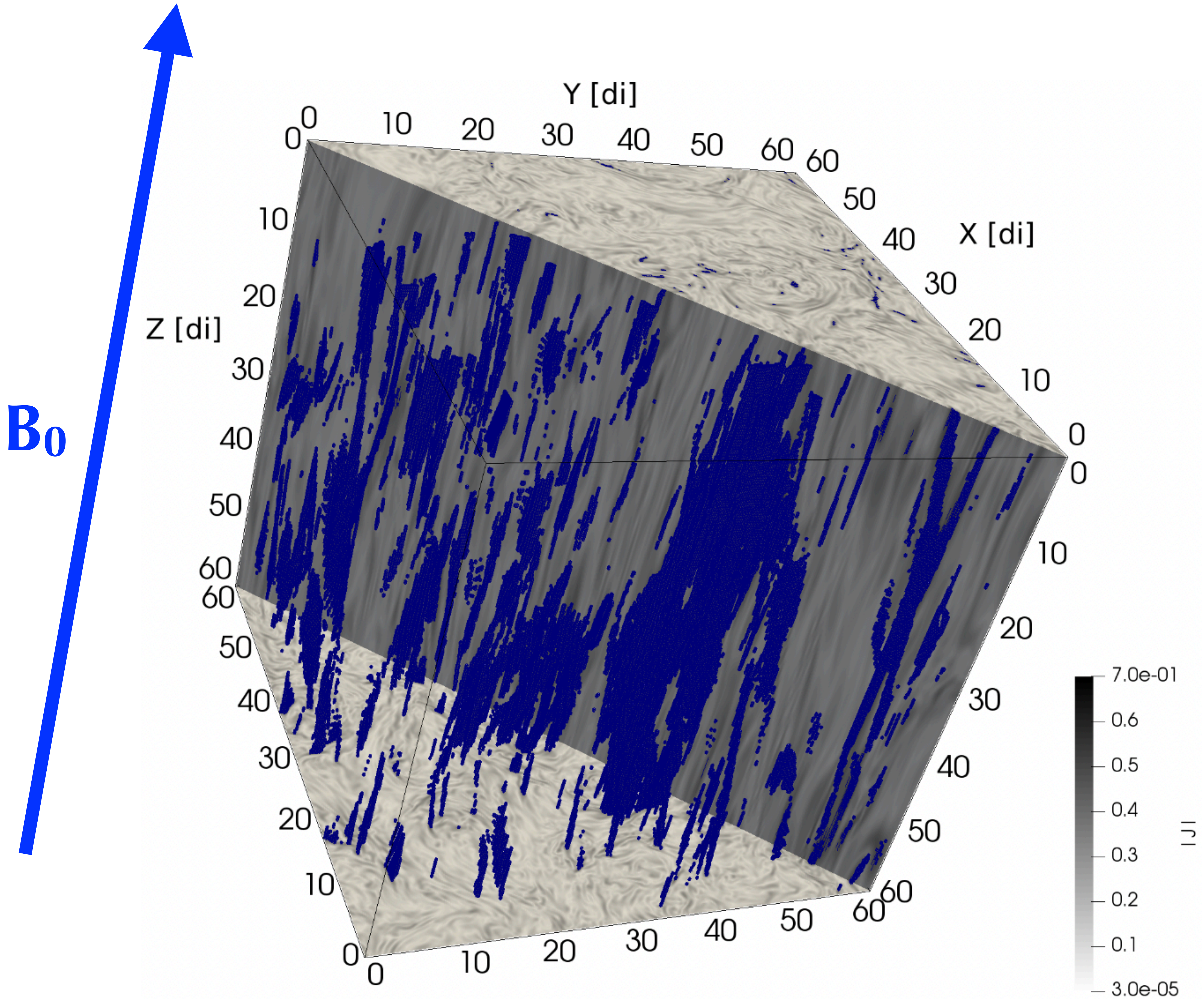
☞ Immediate visual difference: *anisotropy of structures*

hydrodynamic turbulence ($\mathbf{B} = 0$)



[Goto, PTPS 2012]

MHD turbulence ($\mathbf{B} \neq 0$)



[Sisti et al., A&A 2021]

Alfvénic Turbulence

The MHD equations in the Elsässer formulation

$$\frac{\partial \mathbf{z}^+}{\partial t} + \mathbf{z}^- \cdot \nabla \mathbf{z}^+ = -\nabla P$$

$$\frac{\partial \mathbf{z}^-}{\partial t} + \mathbf{z}^+ \cdot \nabla \mathbf{z}^- = -\nabla P$$

$$\boldsymbol{z}^{\pm} = \boldsymbol{u} \pm \frac{\boldsymbol{B}}{\sqrt{4\pi\varrho}}$$

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Alfvén waves traveling "up" or "down" the magnetic field **B**

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non-linear interaction only between counter-propagating Alfvén waves

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Alfvén waves traveling "up" or "down" the magnetic field **B**

Alfvénic turbulence ~ interaction of counter-propagating AWs

Alfvénic Turbulence

Split background and fluctuations:

$$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{b}$$

$$\mathbf{u} = \cancel{\mathbf{u}}_0 + \delta\mathbf{u}$$

$$\mathbf{v}_{\mathrm{A}} = \frac{\mathbf{B}_0}{\sqrt{4\pi\varrho_0}}$$

$$\delta\mathbf{z}^{\pm} = \delta\mathbf{u} \pm \frac{\delta\mathbf{b}}{4\pi\varrho_0}$$

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$$\Rightarrow \quad \left(\frac{\partial}{\partial t} \mp \mathbf{v}_\mathrm{A}\cdot\boldsymbol{\nabla}\right)\delta\mathbf{z}^\pm + (\delta\mathbf{z}^\mp\cdot\boldsymbol{\nabla})\delta\mathbf{z}^\pm = \ \ldots \ \text{(!) turbulence needs finite dissipation!}$$

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⇒ non-linearity parameter: $\chi \doteq \frac{\omega_\mathrm{nl}}{\omega_\mathrm{A}} = \frac{k_\perp \delta z^\mp}{k_\parallel v_\mathrm{A}}$

- << 1 ("WEAK")
- ~ 1 ("STRONG")

Phenomenology of Alfvénic Turbulence

weak Alfvénic turbulence: a quick phenomenological derivation of the spectrum

☞ *for a formal derivation, see, e.g.,*

[Ng & Bhattacharjee, PoP 1996]
[Galtier, Nazarenko, Newell, Pouquet, JPP 2000]
[Schekochihin, arXiv:2010.00699]

(because, yes, it's the last talk on Friday and we all want to go to lunch!)

Phenomenology of Alfvénic Turbulence

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$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$$
$$\omega(k_{\|,1}) + \omega(k_{\|,2}) = \omega(k_{\|,3})$$

$\Rightarrow$ *no parallel cascade ($k_{//}$ = cst.), only a cascade in $k_\perp$!*

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(change during one collision)

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(change during one collision)

⇒ assume changes accumulates as a random walk: $N_{\text{inter.}} \sim \left(\frac{\delta z}{\Delta(\delta z)}\right)^2 \sim \frac{1}{\chi^2}$

$$\Rightarrow \quad \tau_{casc} \sim N\,\tau_A \sim \frac{\tau_{nl}^2}{\tau_A} = \frac{\tau_{nl}}{\chi}$$

CASCADE TIME

Phenomenology of Alfvénic Turbulence

weak Alfvénic turbulence: a quick phenomenological derivation of the spectrum

☞ fluctuations' scaling and energy spectrum from constant energy flux through scales:

$$\frac{\delta z^2}{\tau_{\rm casc}} \sim \varepsilon = \text{const.} \quad \Rightarrow \quad \delta z \propto k_\perp^{-1/2} \quad \Rightarrow \quad \mathcal{E}_{\delta z} \propto k_\perp^{-2}$$

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while linear frequency is constant because there is no parallel cascade:

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$$\Rightarrow \quad \chi \sim k_\perp^{1/2} \quad \Rightarrow \quad \boxed{\frac{\lambda_\perp^{\rm CB}}{\ell_{\|,0}} \sim \left(\frac{\varepsilon\, \ell_{\|,0}}{v_{\rm A}^3}\right)^{1/2} \sim \left(\frac{\delta z_0}{v_{\rm A}}\right)^{3/2} \approx \chi_0^{3/2}} \; (\ll 1)$$

transition to critical balance (χ ~ 1)

Phenomenology of Alfvénic Turbulence

critically balanced (strong) Alfvénic turbulence: a quick phenomenological derivation

☞ *for furhter details, see, e.g.,*

[Goldreich & Sridhar, ApJ 1995]
[Oughton & Matthaeus, ApJ 2020]
[Schekochihin, arXiv:2010.00699]

(same reason as before)

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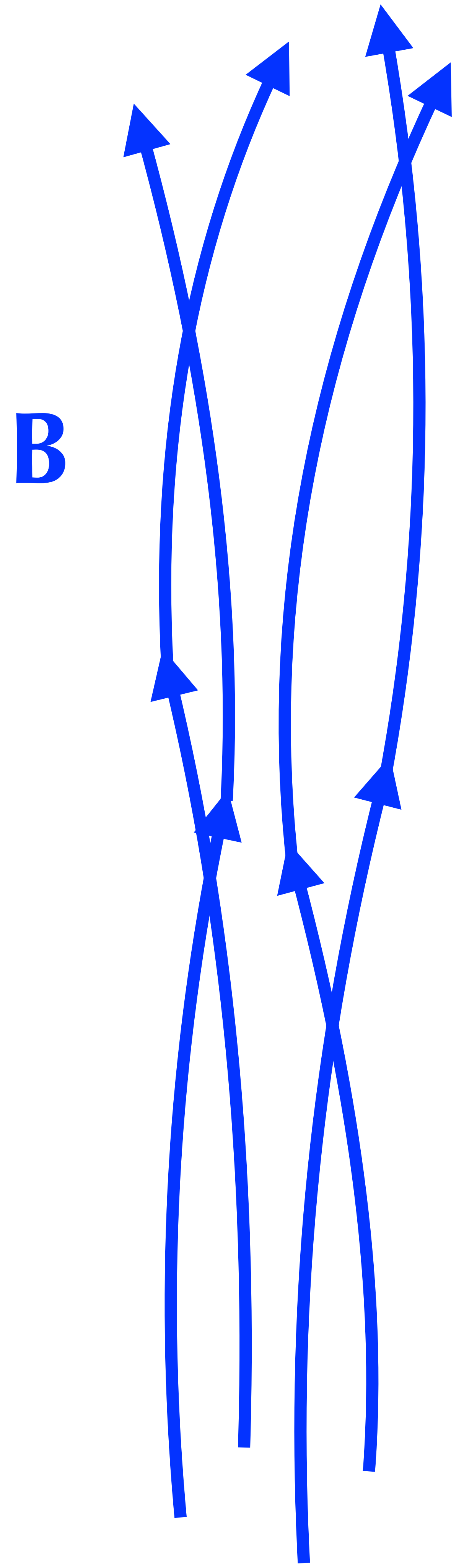

☞ At this point, linear, non-linear, and cascade timescales match each other:

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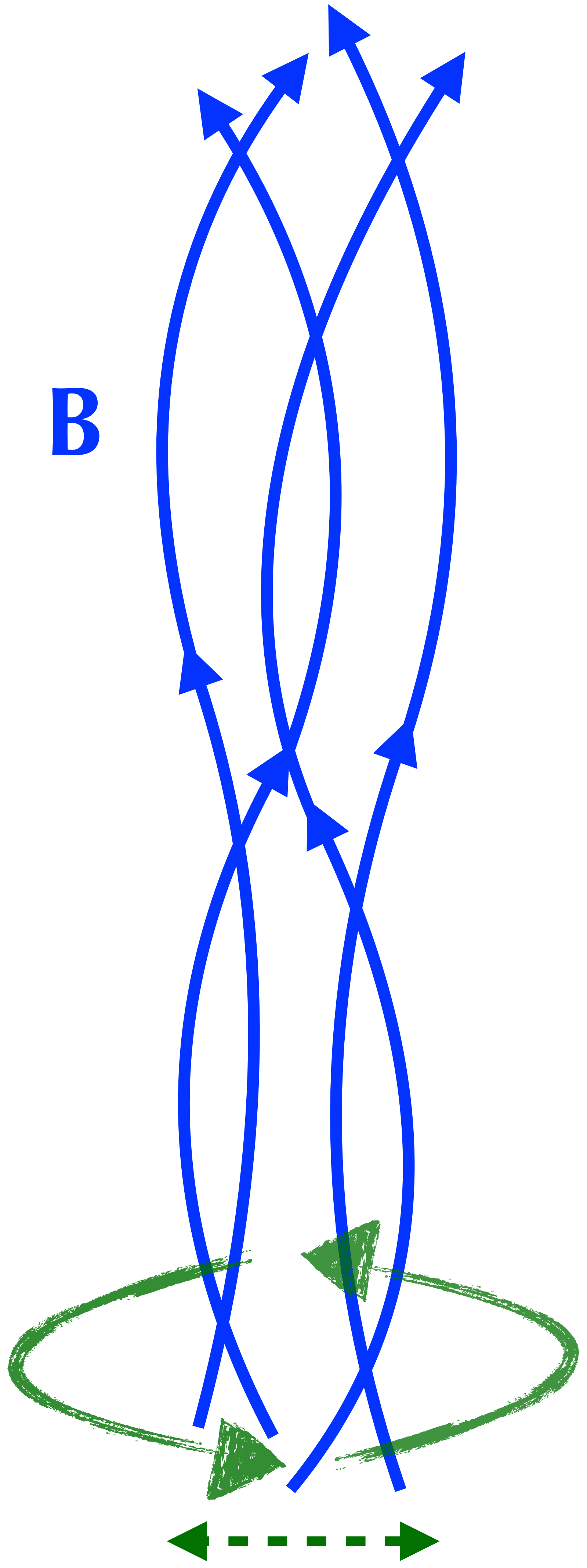

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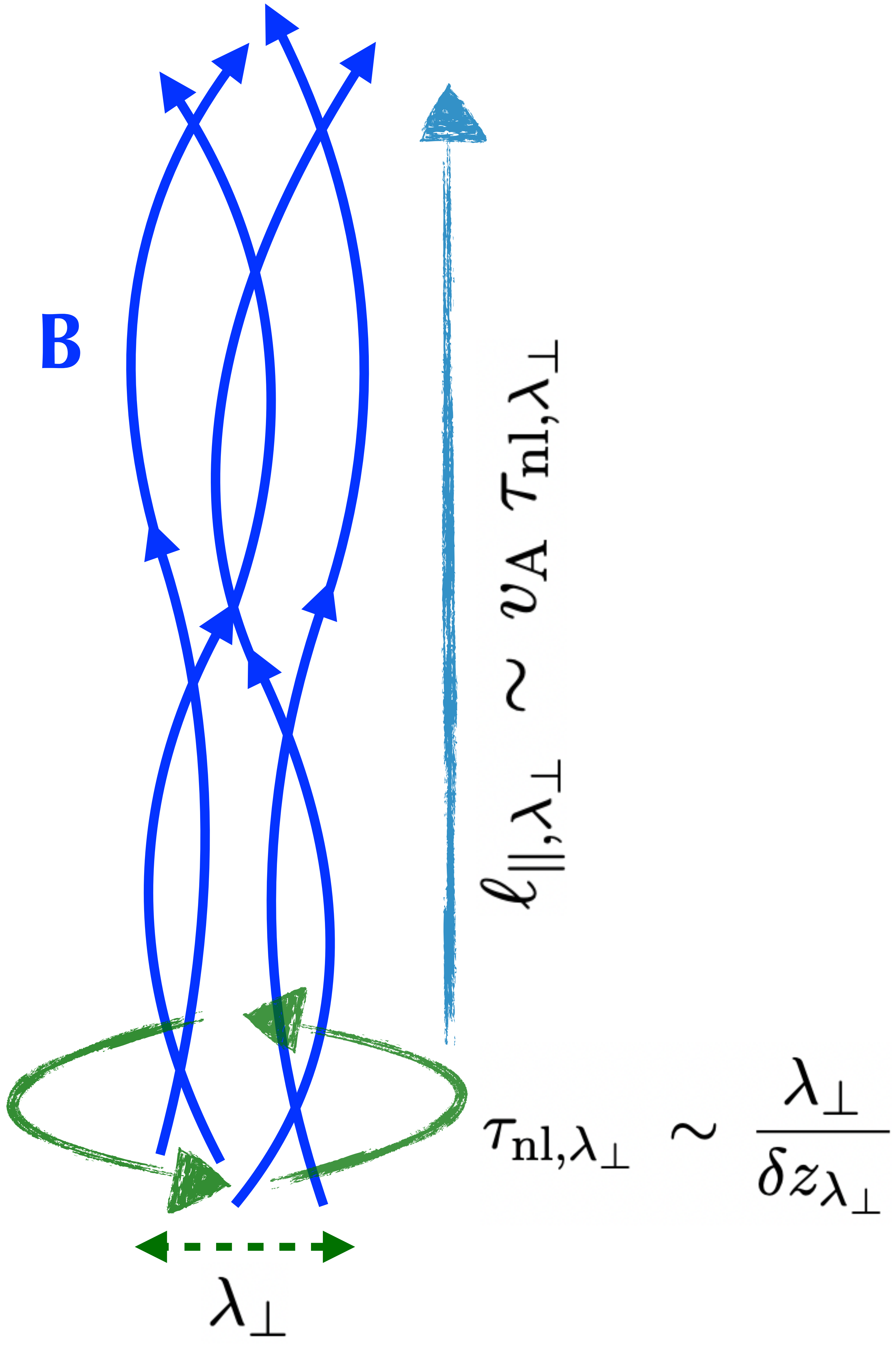
you can see the ***``critical-balance condition" as the result of causality***:

$$\tau_{{\rm nl},\lambda_\perp} \sim \frac{\lambda_\perp}{\delta z_{\lambda_\perp}}$$

$\lambda_\perp$

Phenomenology of Alfvénic Turbulence

critically balanced (strong) Alfvénic turbulence: a quick phenomenological derivation



☞ At this point, linear, non-linear, and cascade timescales match each other:

$$\tau_{nl} \sim \tau_A \quad \Rightarrow \quad \tau_{casc} \sim \tau_{nl}$$

you can see the ***``critical-balance condition" as the result of causality***:

the information about Alfvénic fluctuations decorrelating in the perpendicular plane over an eddy turn-over time τ_{nl} can only propagate along the field for a length $\ell_{||}$ at maximum speed v_A.

"So… CB is essentially AWs trying to keep up with the turbulent eddies…"

Phenomenology of Alfvénic Turbulence

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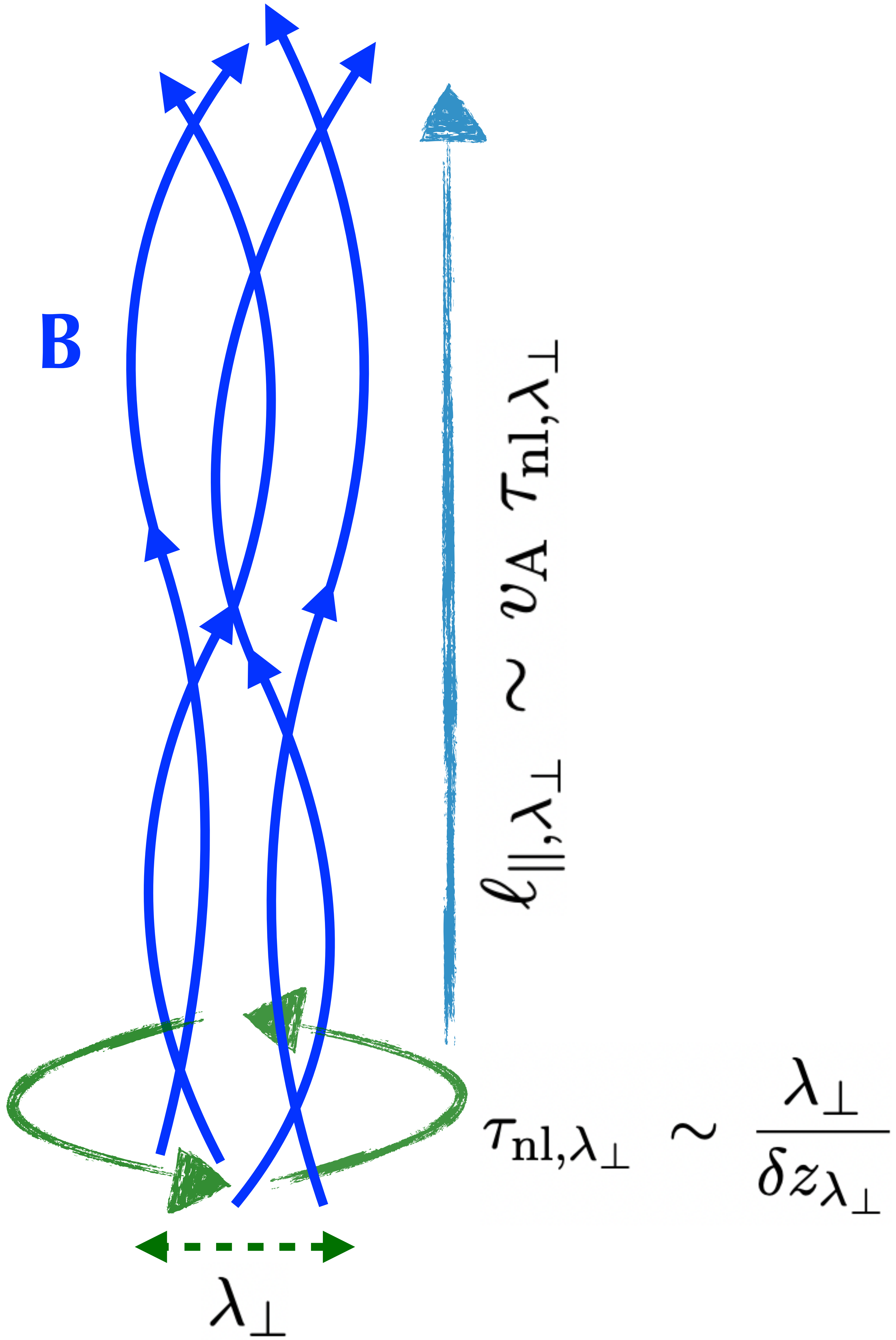

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"So… CB is essentially AWs trying to keep up with the turbulent eddies…"

Therefore, once $\tau_{nl} \sim \tau_A$ is reached, the balance is mantained.

(In principle, this could be done by continuing the cascade with τ_{nl} = const., or by generating smaller $\ell_{||}$ such that $\tau_A \sim \ell_{||}/v_A \sim \tau_{nl}$ keeps holding… it is the latter)

Phenomenology of Alfvénic Turbulence

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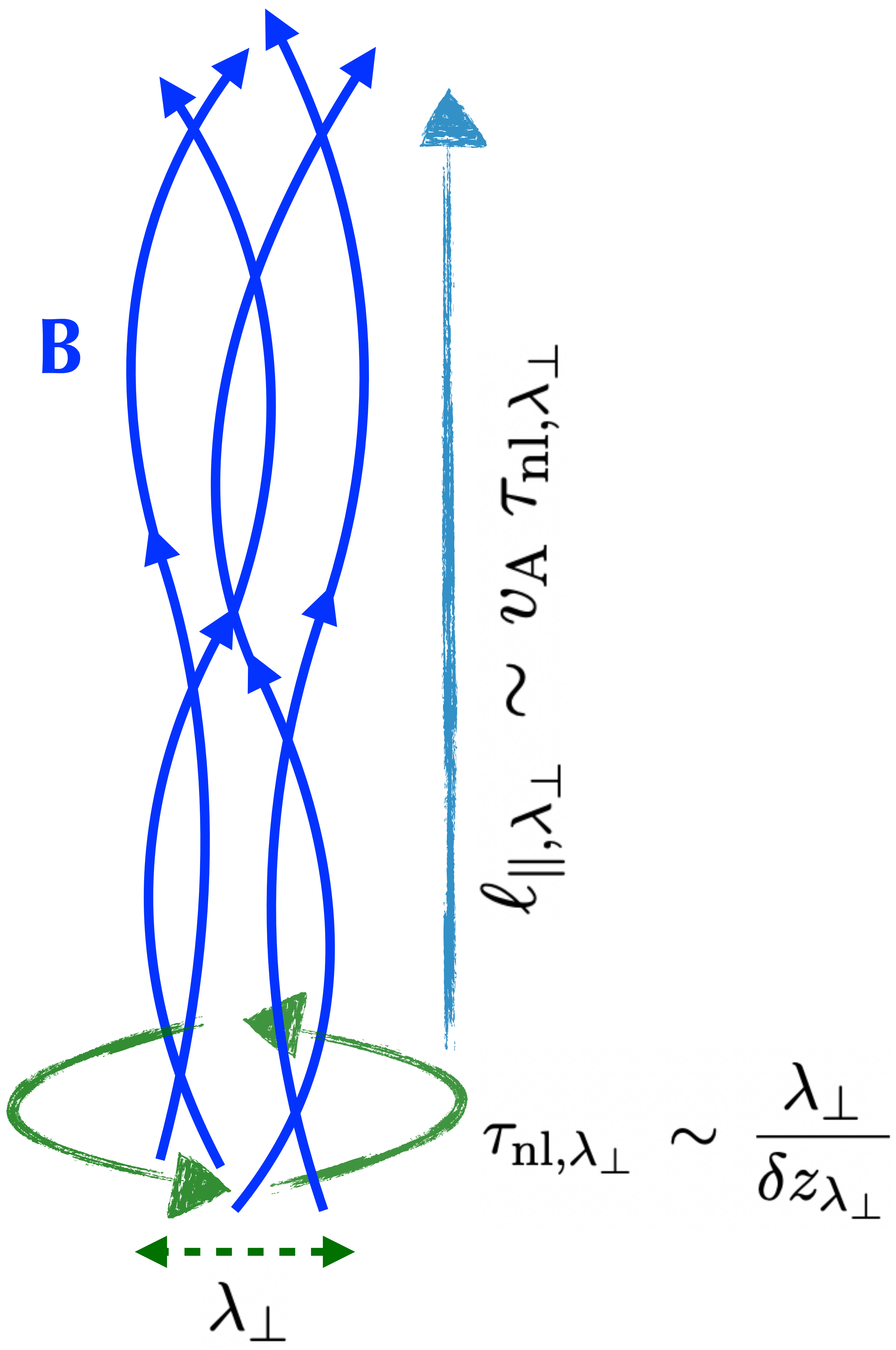

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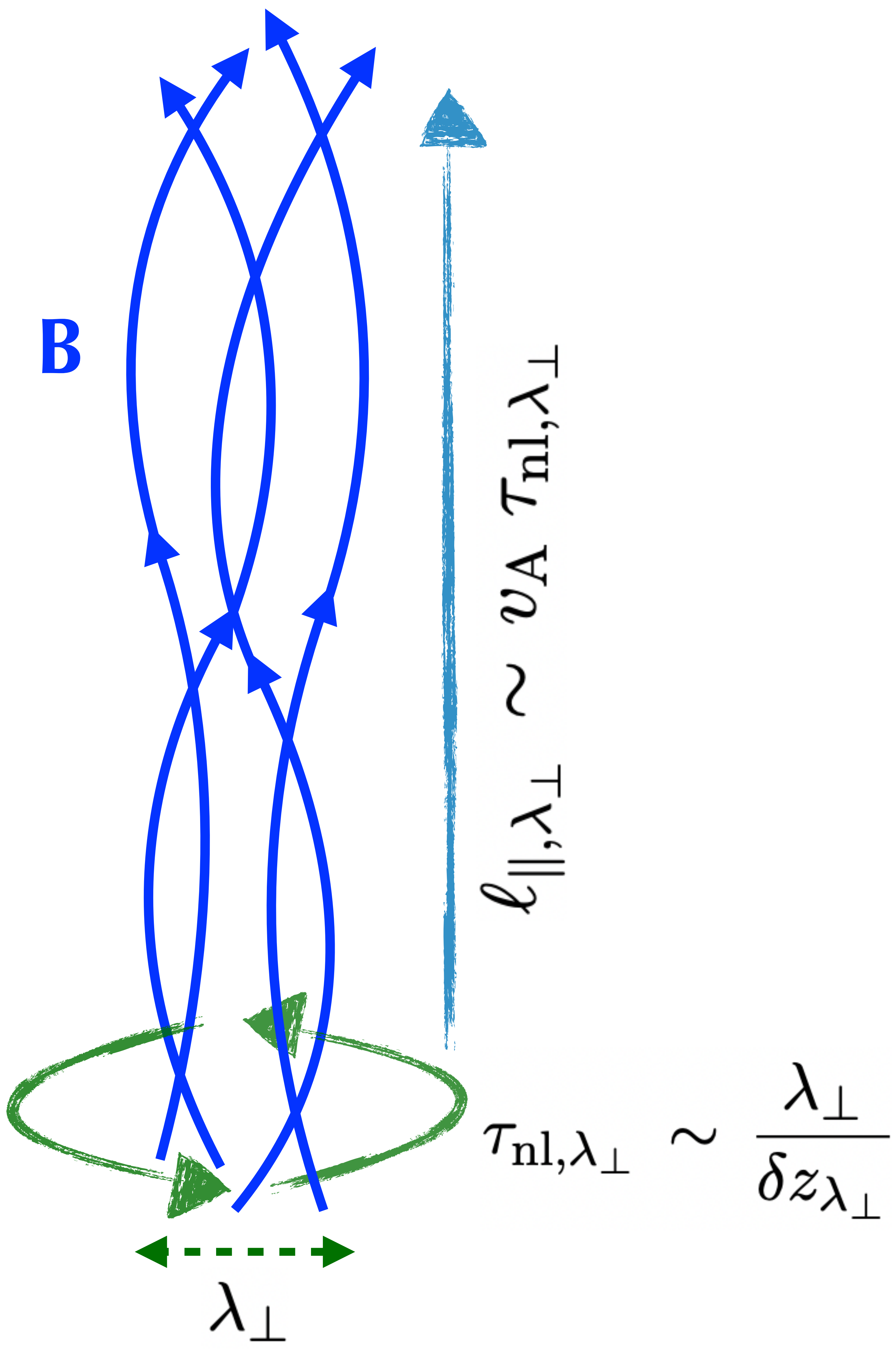

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☞ fluctuations' scaling + spectum from **ε** = const. (*you know the drill*):

$$\frac{\delta z_{k_\perp}^2}{\tau_{\mathrm{nl},k_\perp}} \sim \varepsilon = \mathrm{const.} \quad \Rightarrow \quad \boxed{\delta z_{k_\perp} \propto k_\perp^{-1/3} \quad \Rightarrow \quad \mathcal{E}_{\delta z}(k_\perp) \propto k_\perp^{-5/3}}$$

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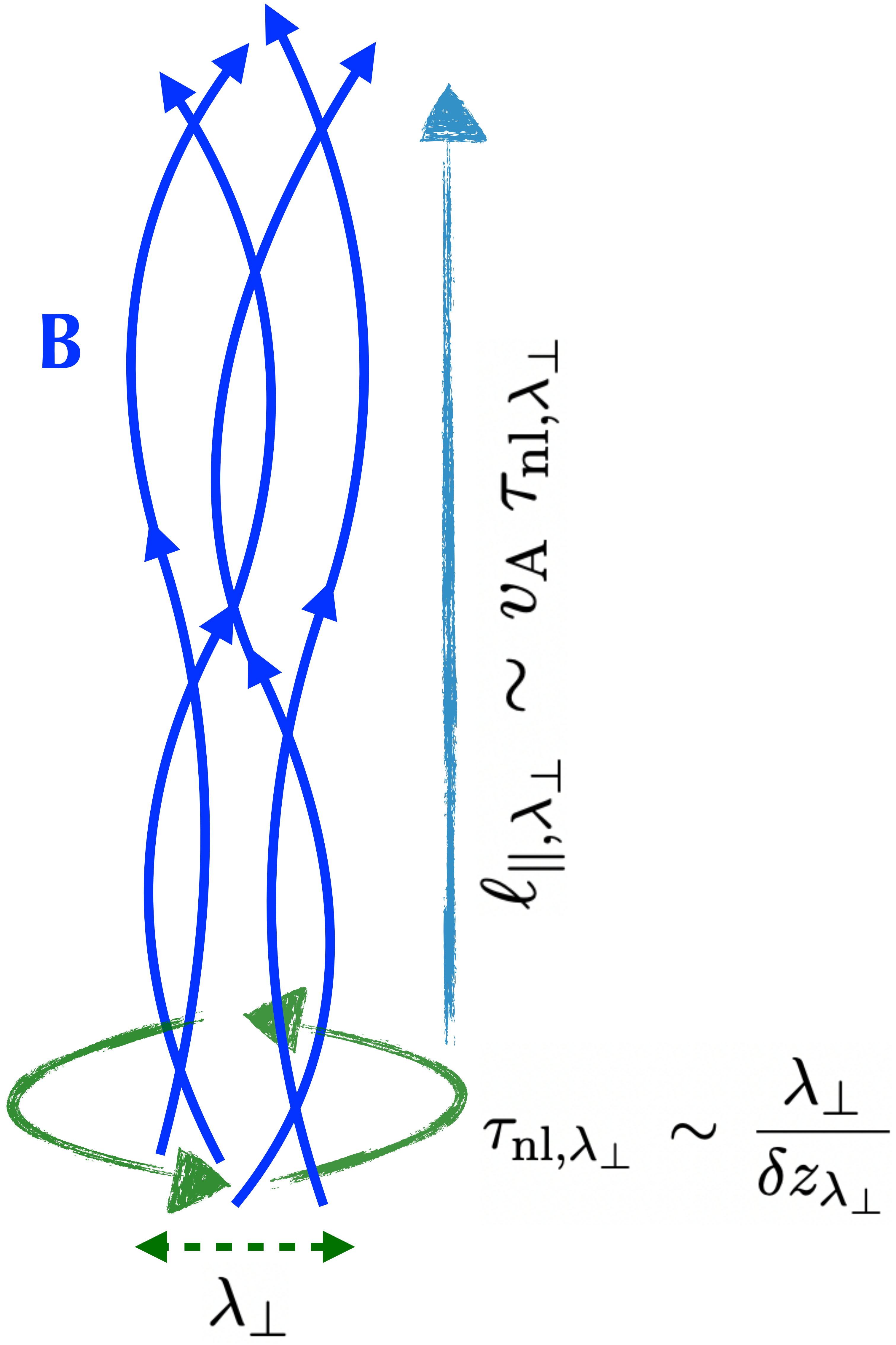

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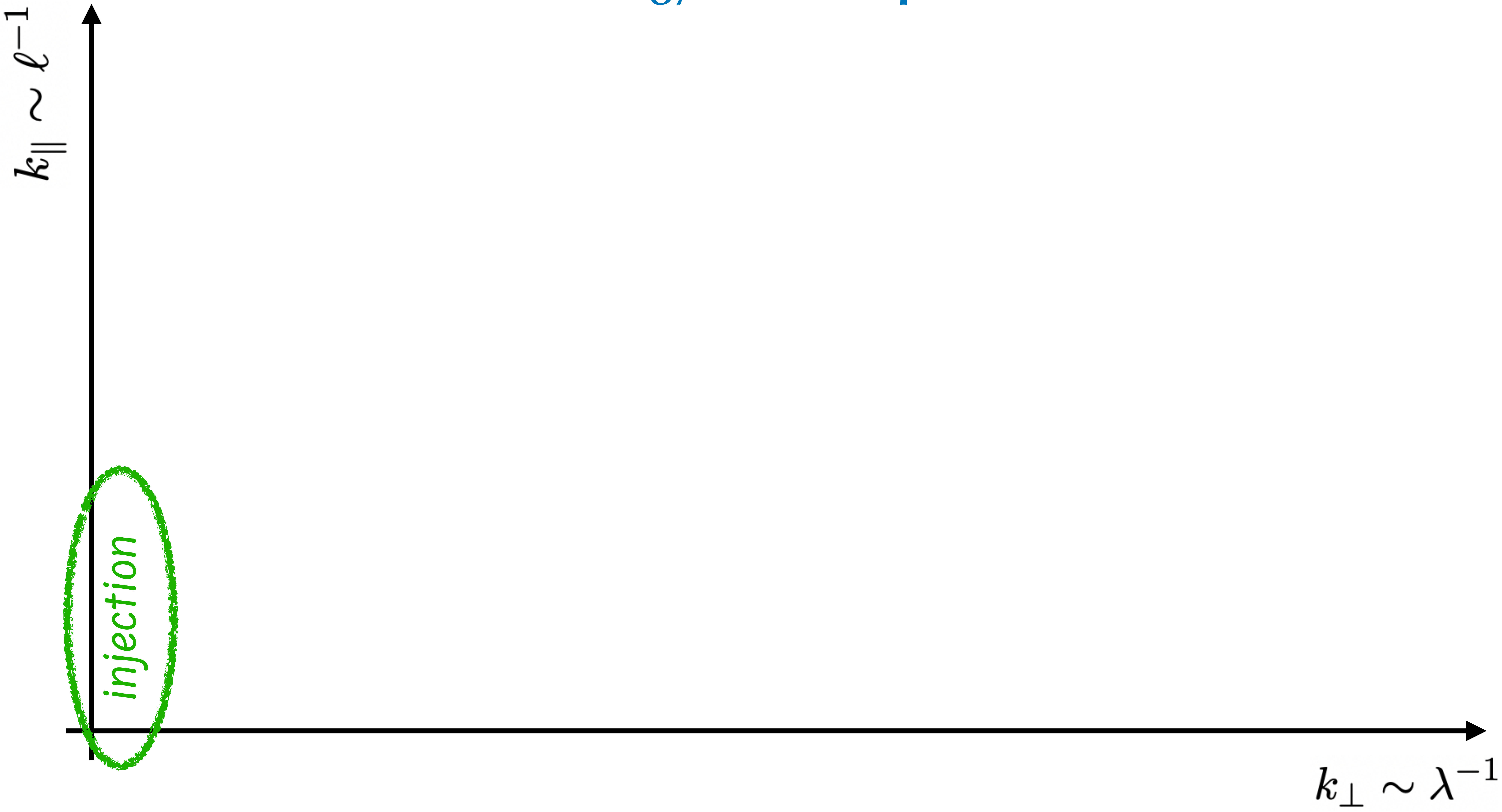
☞ now, you can also compute the fluctuations' wavenumber anisotropy:

$$k_\perp \delta z_{k_\perp} \sim k_\| v_A \quad \Rightarrow \quad \boxed{k_\| \propto k_\perp^{2/3}} \quad \left(\Rightarrow \quad \mathcal{E}_{\delta z}(k_\|) \propto k_\|^{-2} \right)$$

Phenomenology of Alfvénic Turbulence

Energy flux in k space



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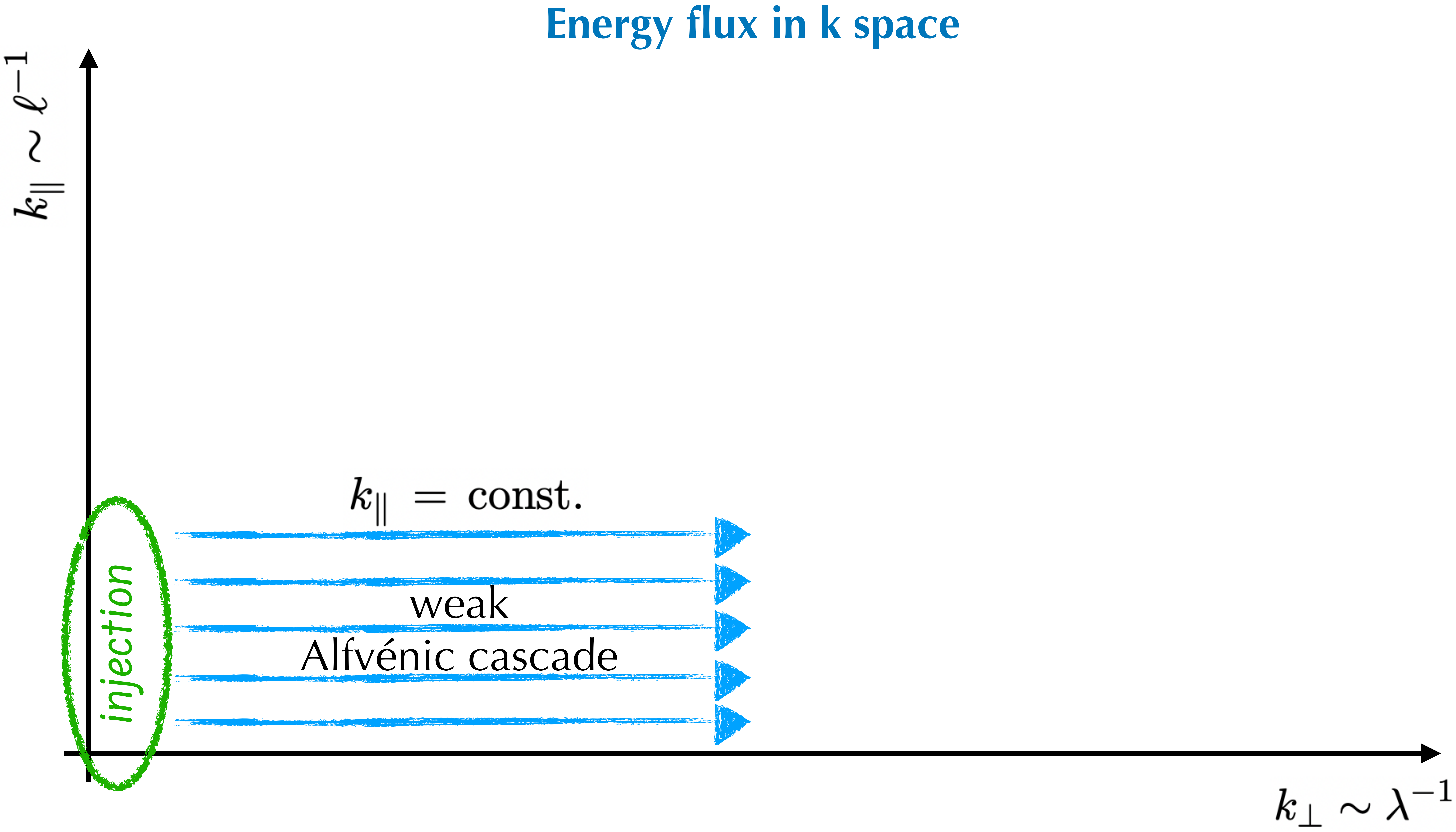

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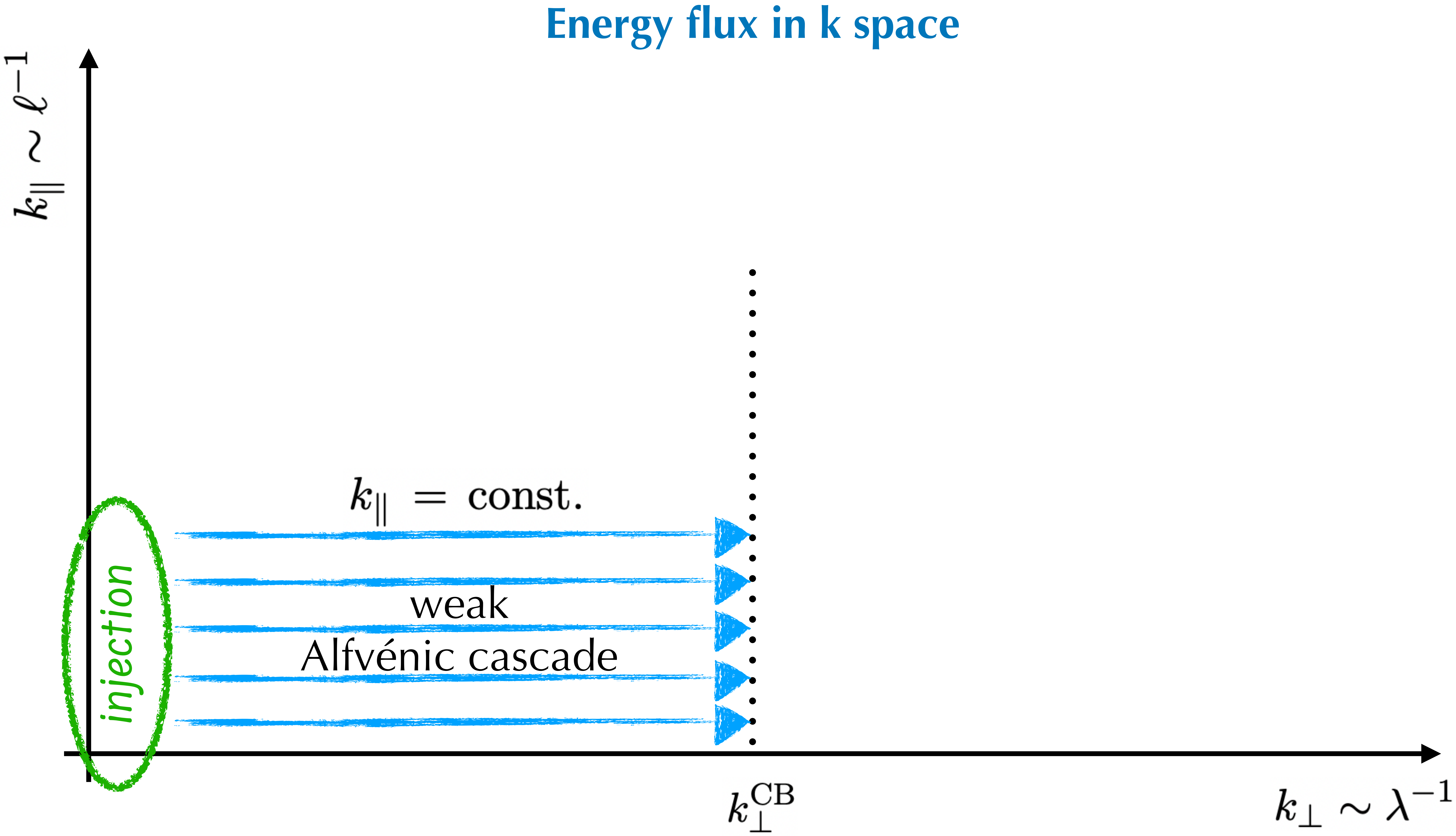

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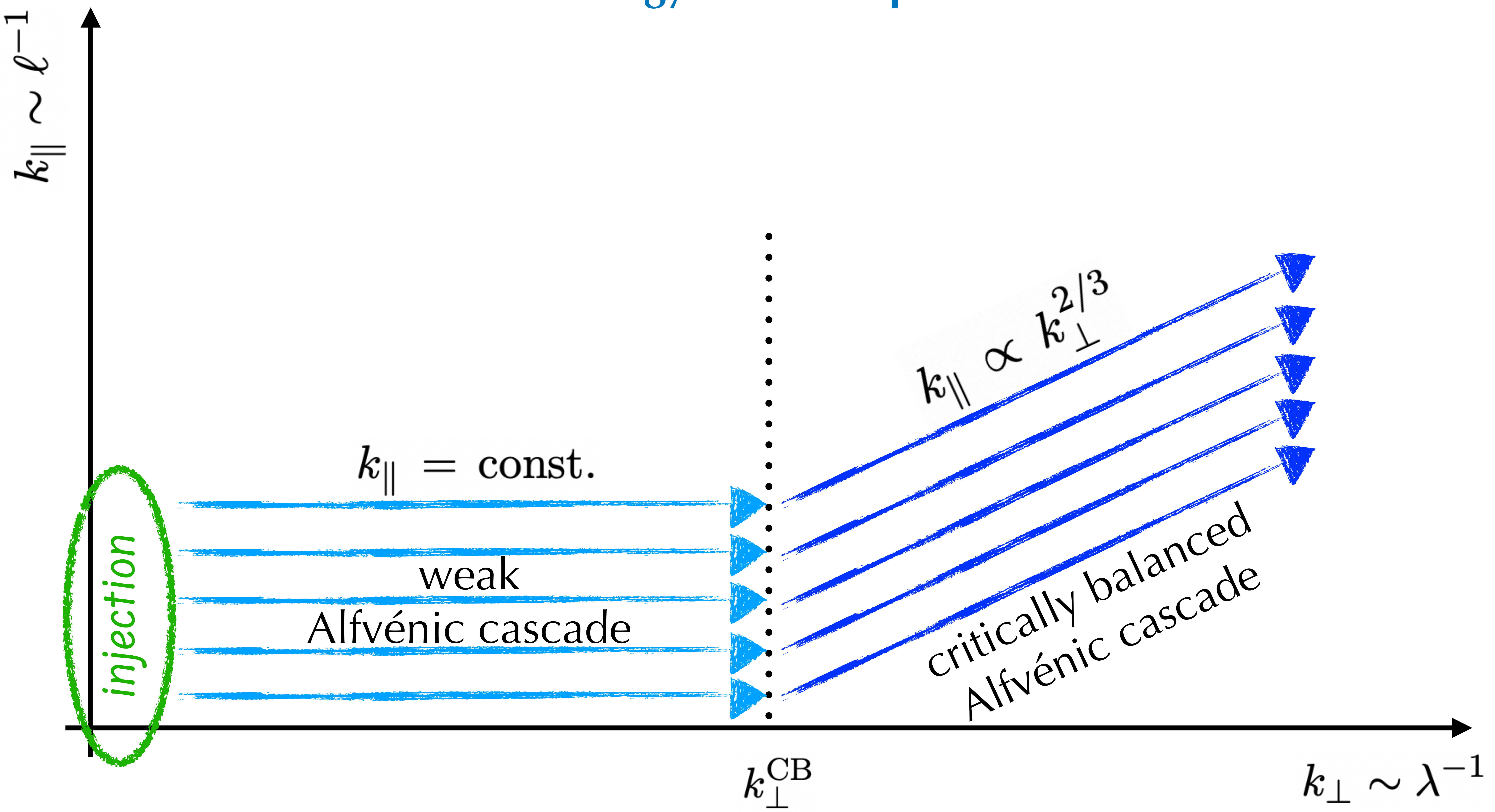

Developments in Theoretical Models

Further Developments in Theoretical Models

dynamic alignment in Alfvénic turbulence: three-dimensional anisotropy

☞ *for furhter details, see, e.g.,*

[Boldyrev, PRL 2006]
[Schekochihin, arXiv:2010.00699]

Further Developments in Theoretical Models

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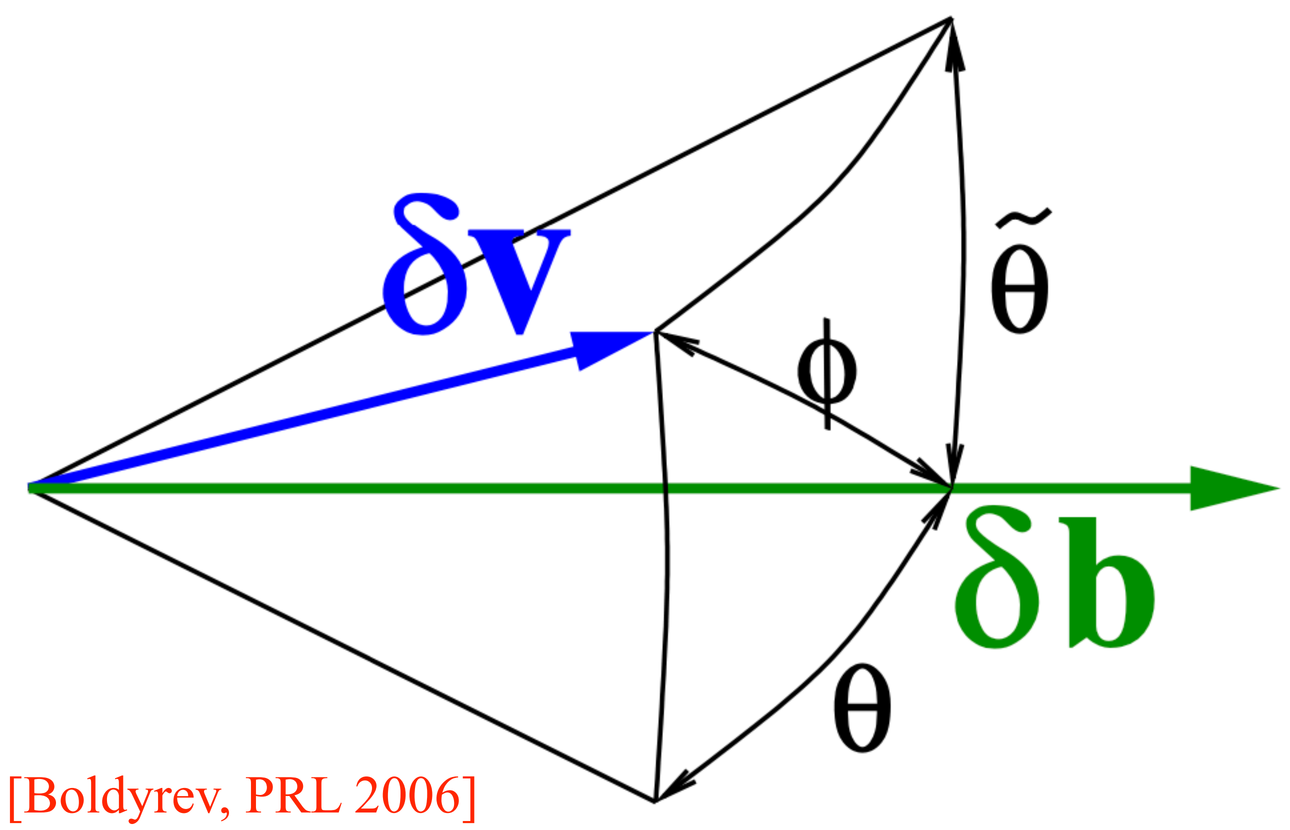

[Boldyrev, PRL 2006]

☞ Observations and simulations show that $\boldsymbol{\delta v}_\lambda$ and $\boldsymbol{\delta b}_\lambda$ have a spontaneous tendency to align in the plane perpendicular to the local mean field $\langle \mathbf{B} \rangle_\lambda$, within an angle θ_λ

(e.g., Podesta et al., JGR 2009; Hnat et al., PRE 2011; Mason et al., ApJ 2011; Wicks et al., PRL 2013; Mallet et al., MNRAS 2016; …)

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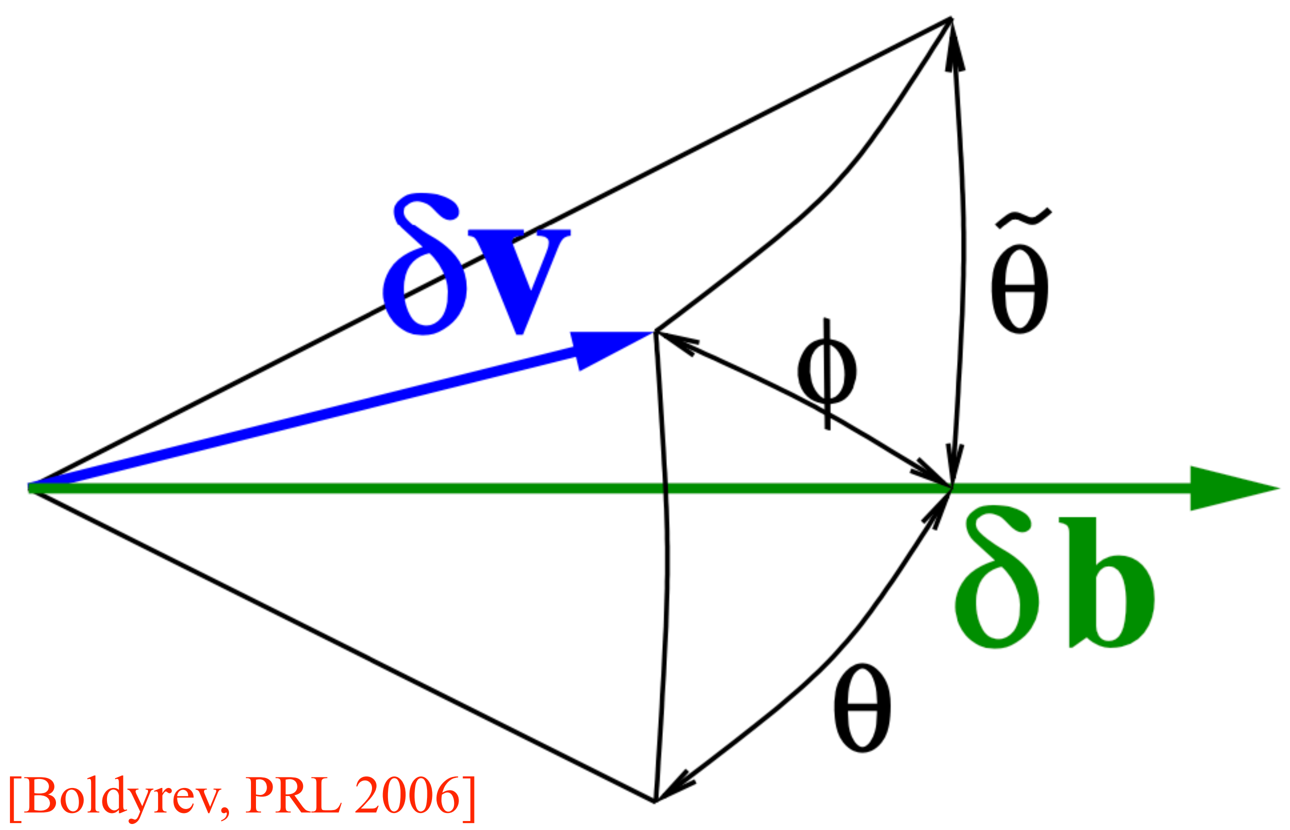

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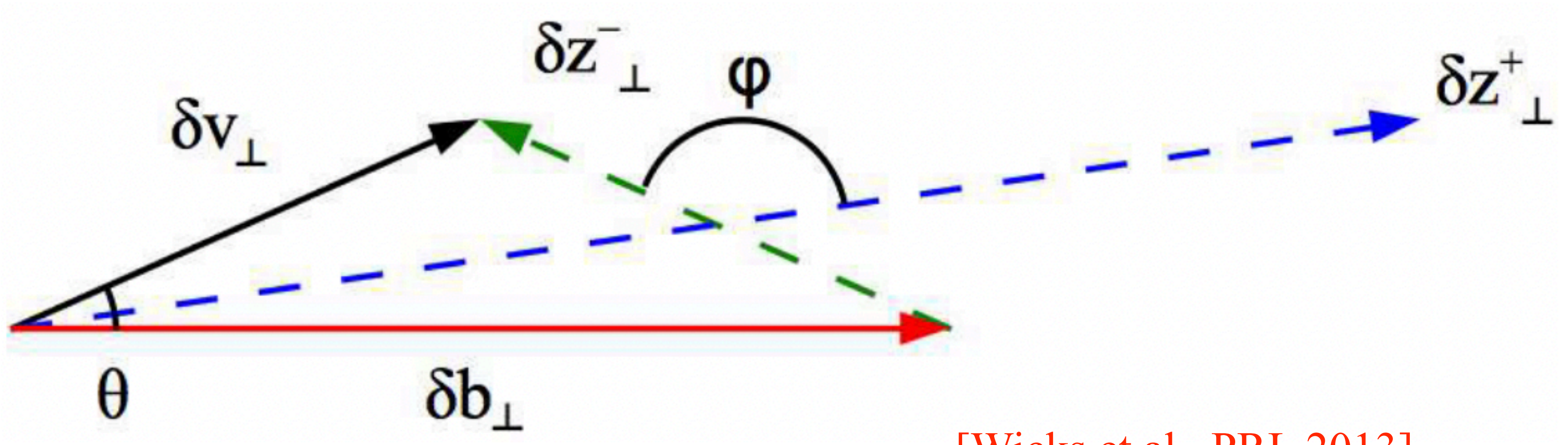
⚠ the alignment between $\mathbf{\delta v}_\lambda$ and $\mathbf{\delta b}_\lambda$ is *not the same* as the alignment between $\mathbf{\delta z}^+_\lambda$ and $\mathbf{\delta z}^-_\lambda$!

(but they are related: see Schekochihin arXiv:2010.00699)



[Wicks et al., PRL 2013]

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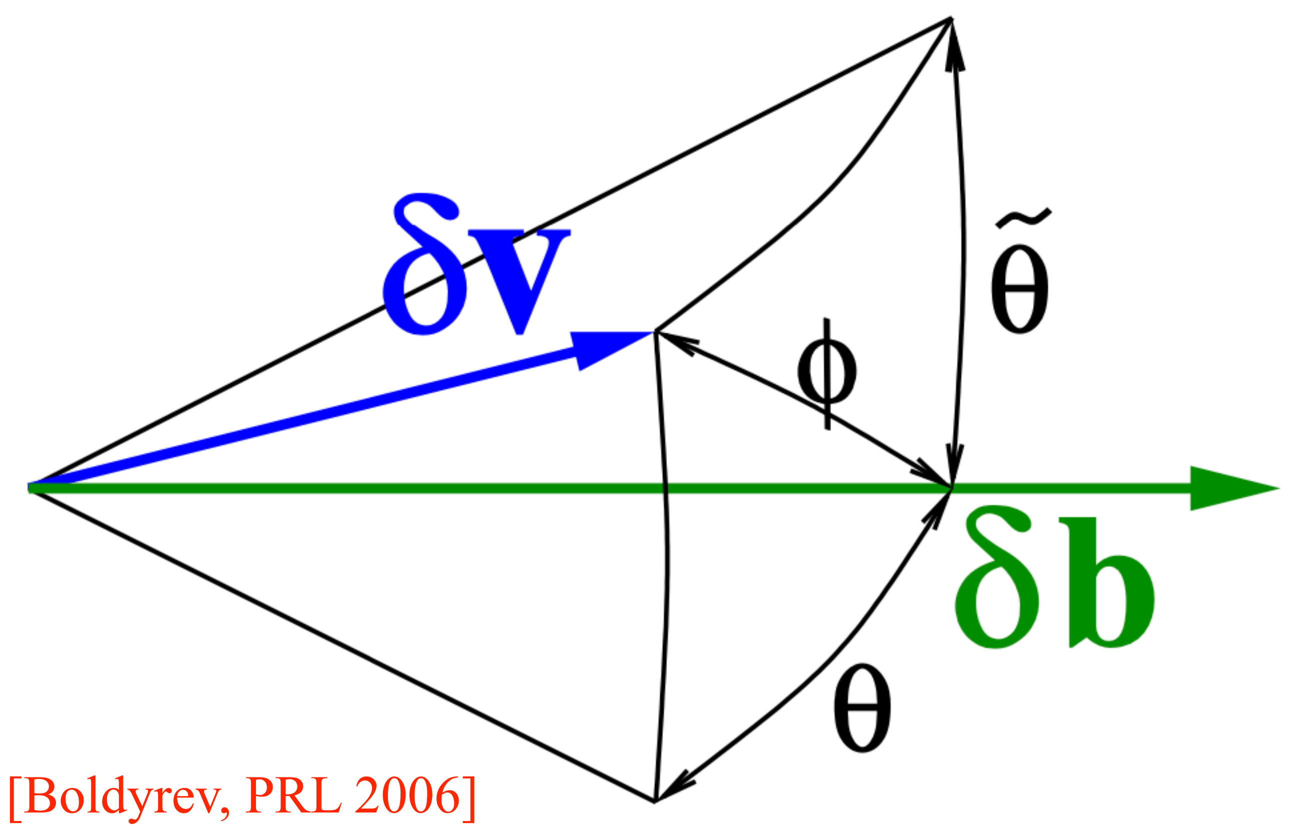

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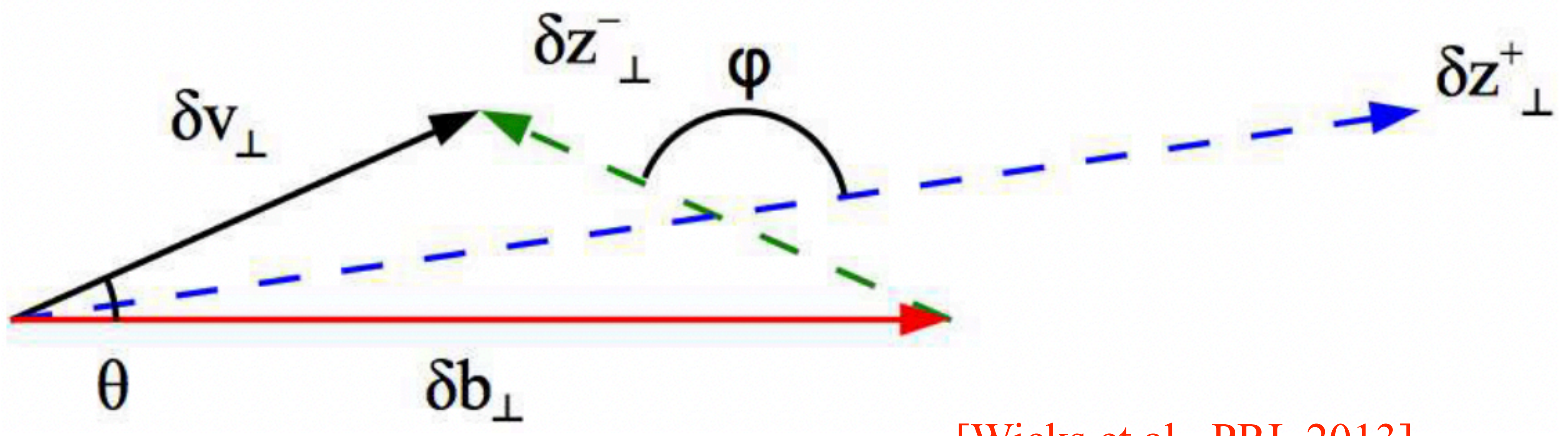


[Wicks et al., PRL 2013]

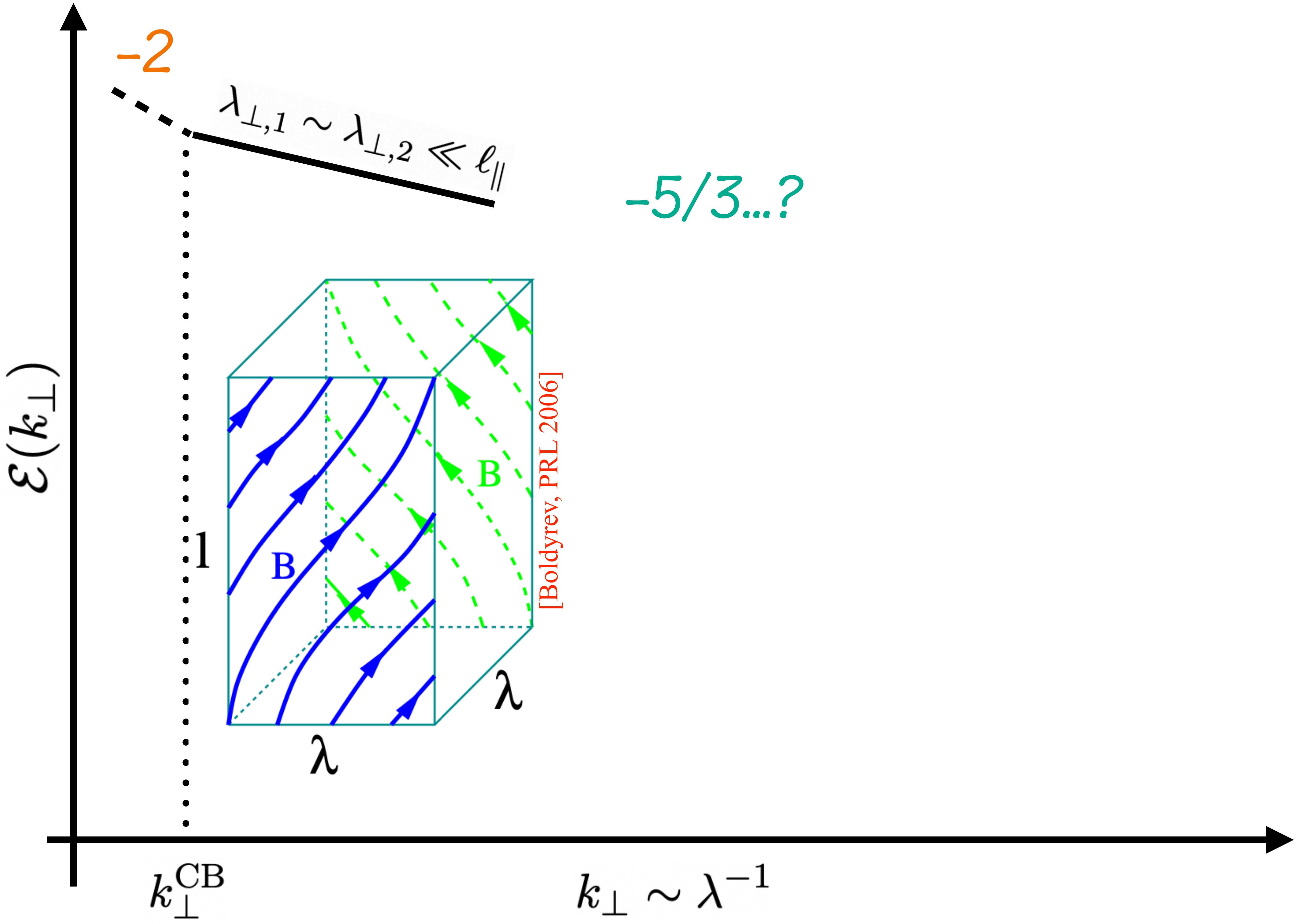
alignment ⇒ depletion of non-linearities: $\delta\mathbf{z}^{\mp}\cdot\boldsymbol{\nabla}\delta\mathbf{z}^{\pm} \sim \sin\varphi_\lambda \dfrac{\delta z_\lambda^2}{\lambda} \approx \varphi_\lambda \dfrac{\delta z_\lambda^2}{\lambda} \quad \longleftrightarrow \quad \theta_\lambda \dfrac{\delta v_\lambda^2}{\lambda}$

⚠ but remember that *fluctuations cannot be perfectly aligned* ($\theta_\lambda = 0$) in order to have a non-linear cascade

Further Developments in Theoretical Models

dynamic alignment in Alfvénic turbulence: three-dimensional anisotropy

The effect of alignment is not only to make the non-linear interactions weaker, but also to induce anisotropy in the plane perpendicular to the magnetic field **B**



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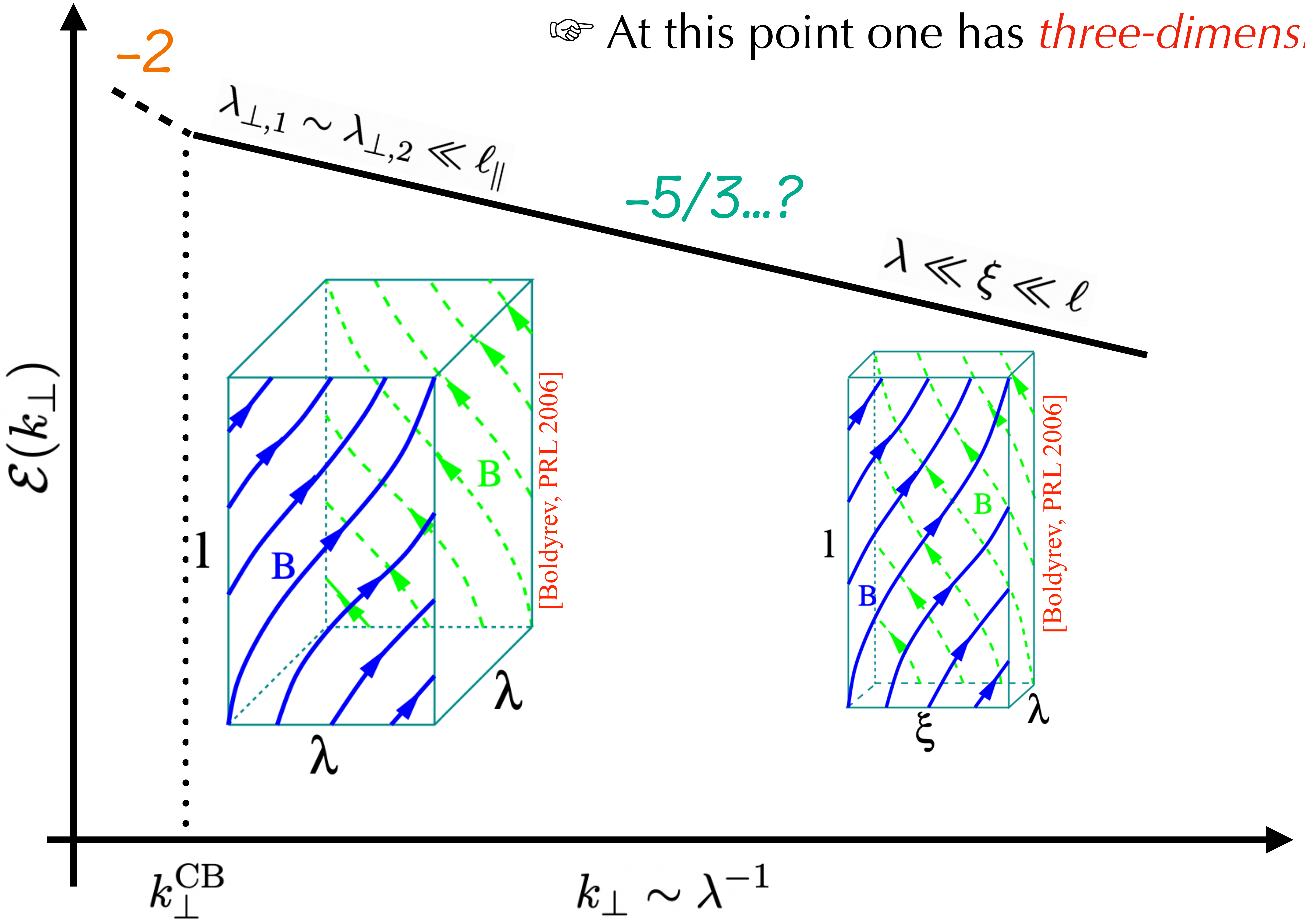

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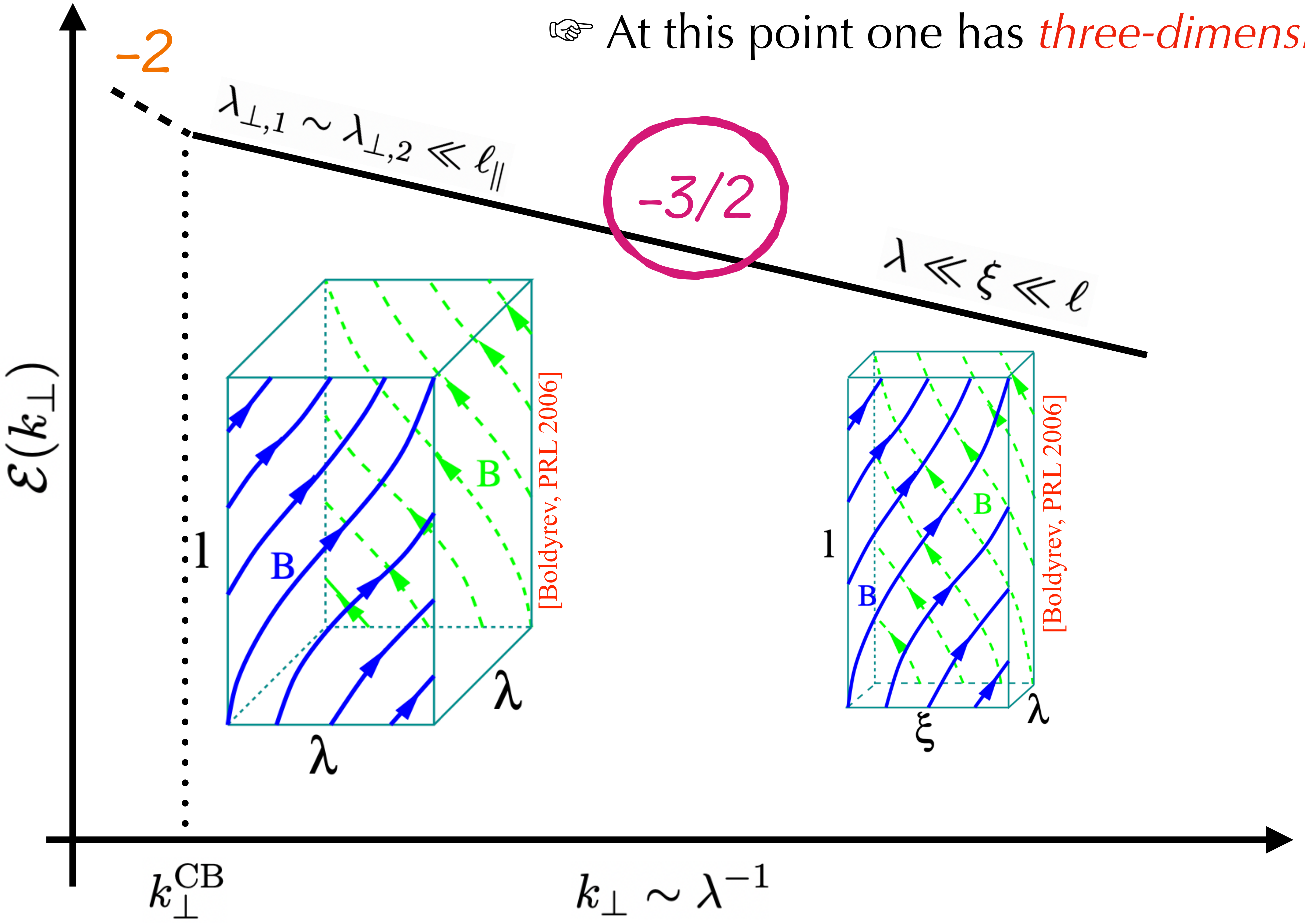

☞ long story short:

(see Boldyrev, PRL 2006 for the derivation)

$$\theta_{k_\perp} \propto k_\perp^{-1/4} \quad \Rightarrow \quad \delta v_{k_\perp} \propto k_\perp^{-1/4} \quad \Rightarrow \quad \mathcal{E}(k_\perp) \propto k_\perp^{-3/2}$$

$$\left(\text{also, now } k_{\|} \propto k_\perp^{1/2}\right)$$

Recent Developments in Theoretical Models

reconnection-mediated regime in Alfvénic turbulence

☞ *for furhter details, see, e.g.,*

[Boldyrev & Loureiro, ApJ 2017]
[Mallet, Schekochihin, Chandran, MNRAS 2017]
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Recent Developments in Theoretical Models

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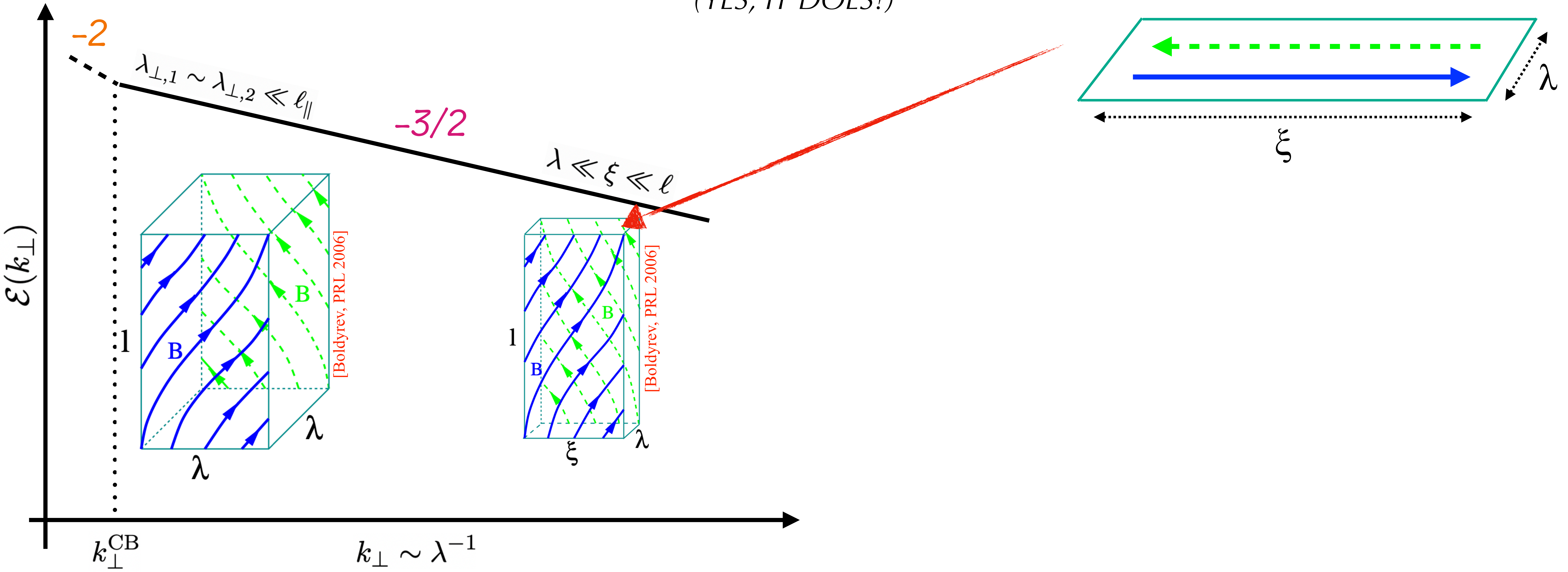
So, we had *three-dimensional anisotropy*, *right?* … wait a minute!

doesn't *3D anisotropy of the turbulent eddies look line a* ***current sheet in the plane perpendicular to B****?!*

(YES, IT DOES!)



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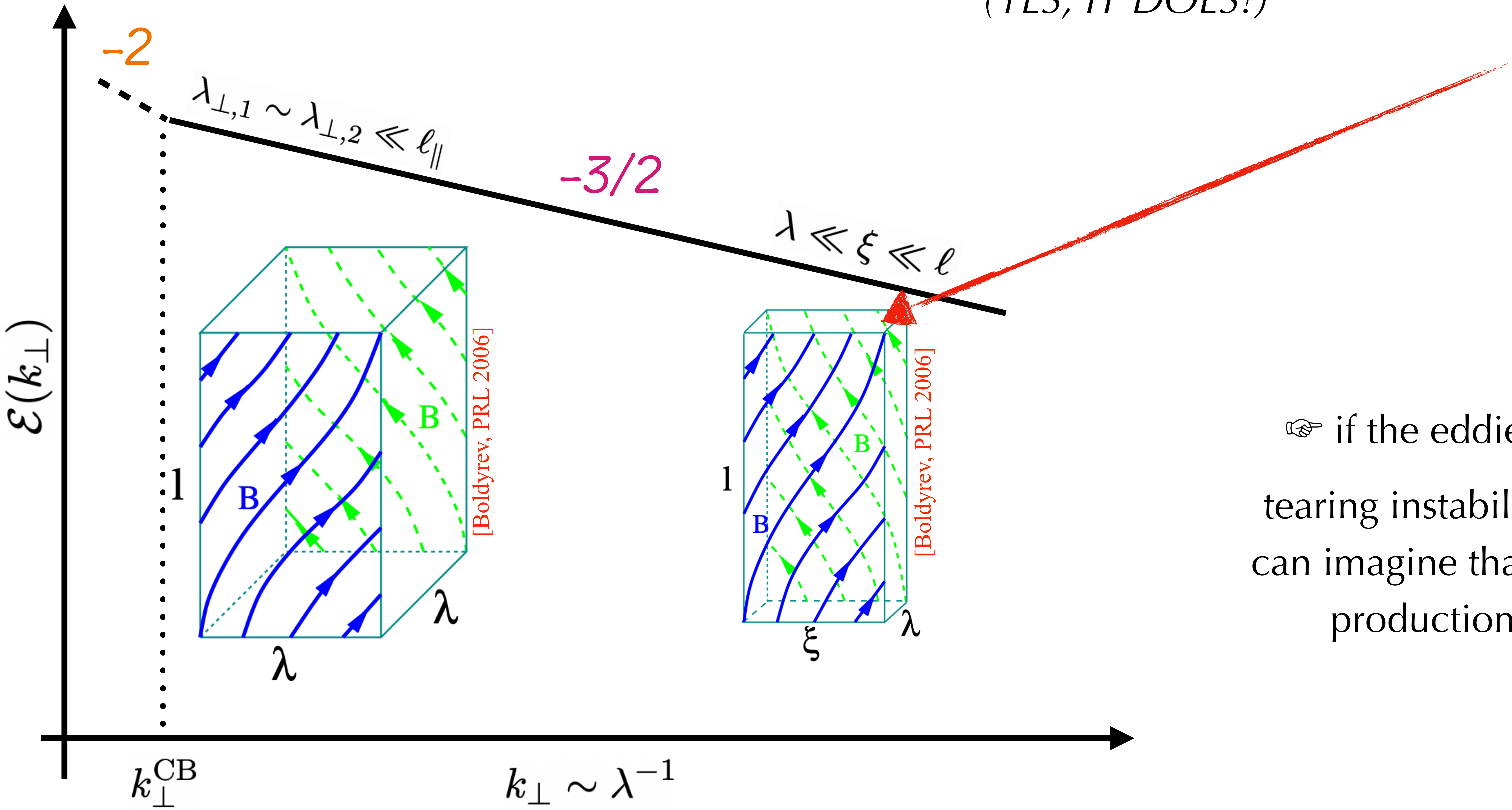

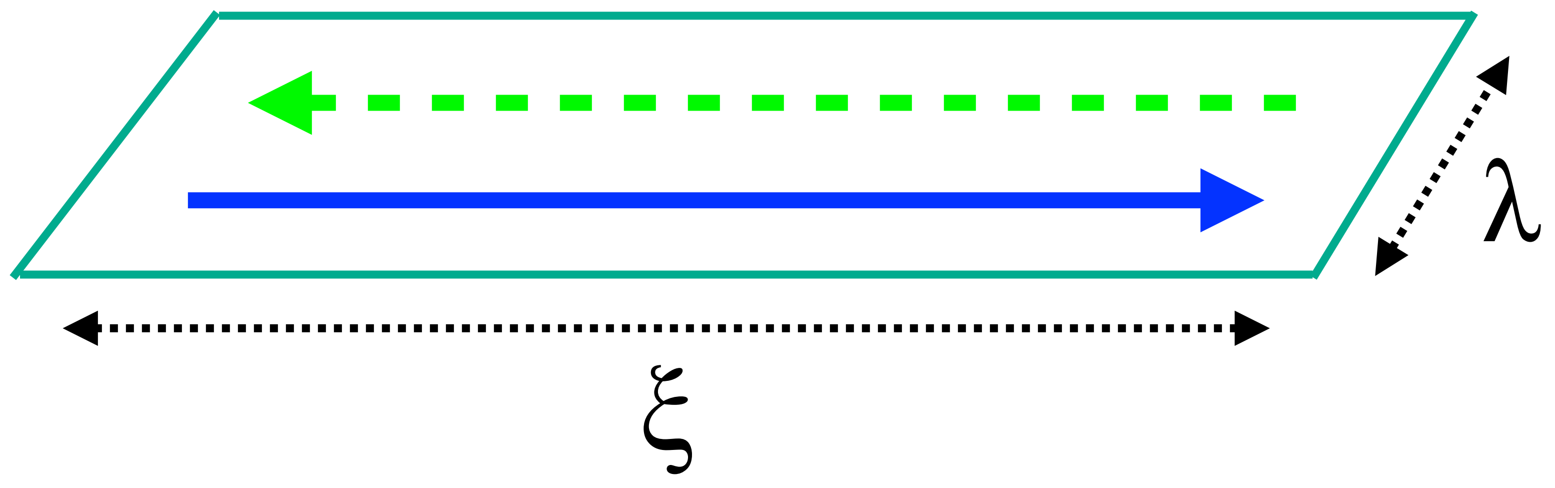

☞ if the eddies at a scale live "long enough" for the tearing instability (i.e., reconnection) to grow, then we can imagine that this process will be responsible for the production of small-scale magnetic fluctations

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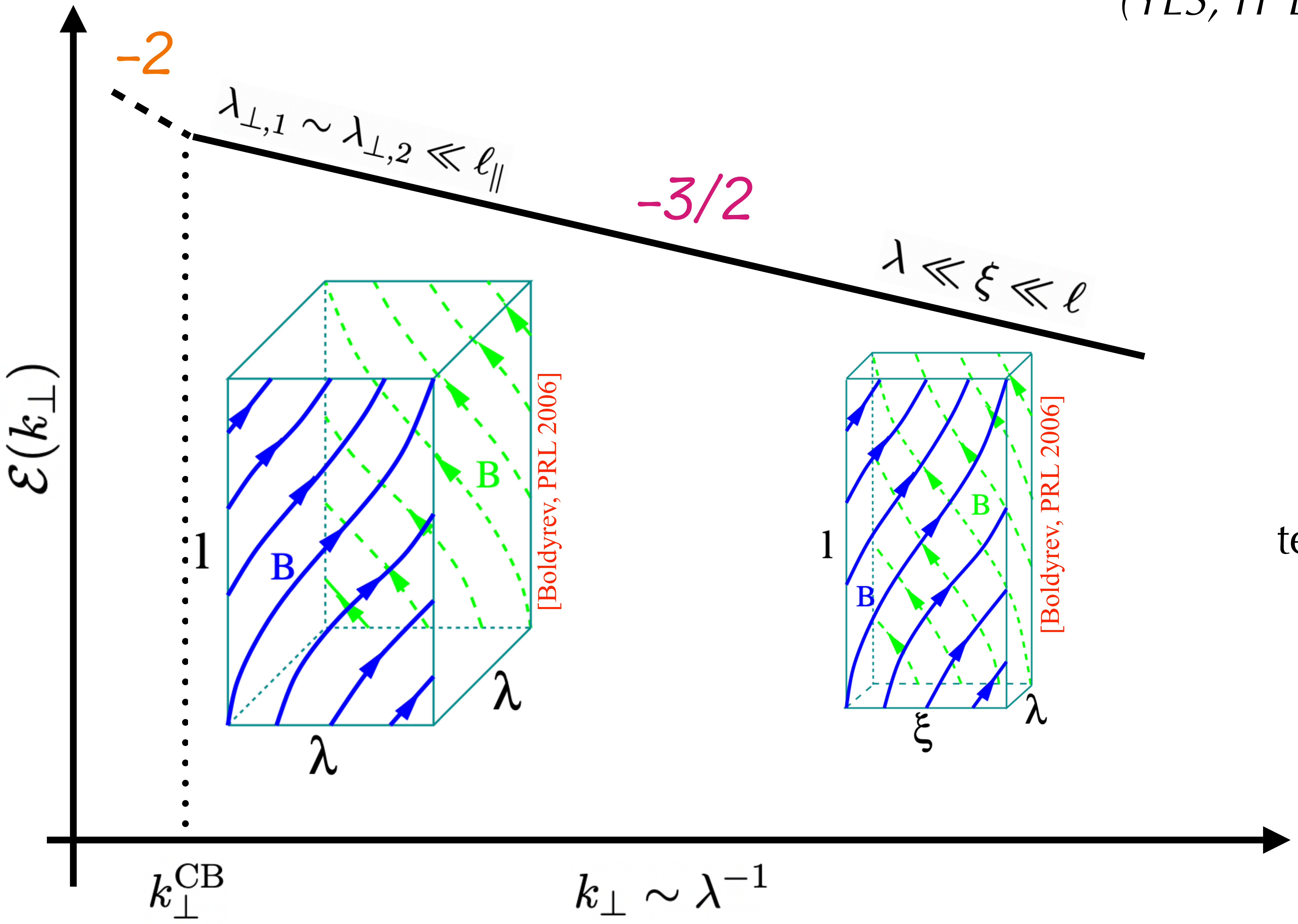

eddy lifetime: $\tau_{{\rm nl},k_\perp} \sim \left(\theta_{k_\perp} k_\perp \delta v_{k_\perp}\right)^{-1} \propto k_\perp^{-1/2}$

tearing growth rate: $\gamma_{k_\perp}^{\rm rec} \sim k_\perp \delta v_{k_\perp} \left(\dfrac{\delta v_{k_\perp}}{k_\perp \eta}\right)^{-1/2} \propto S_0^{-1/2}\, k_\perp^{11/8}$

$$\left(\eta:\ \text{resistivity}\,,\quad S \doteq \frac{v_{\rm A}\,\ell_0}{\eta}:\ \text{Lundquist number}\right)$$

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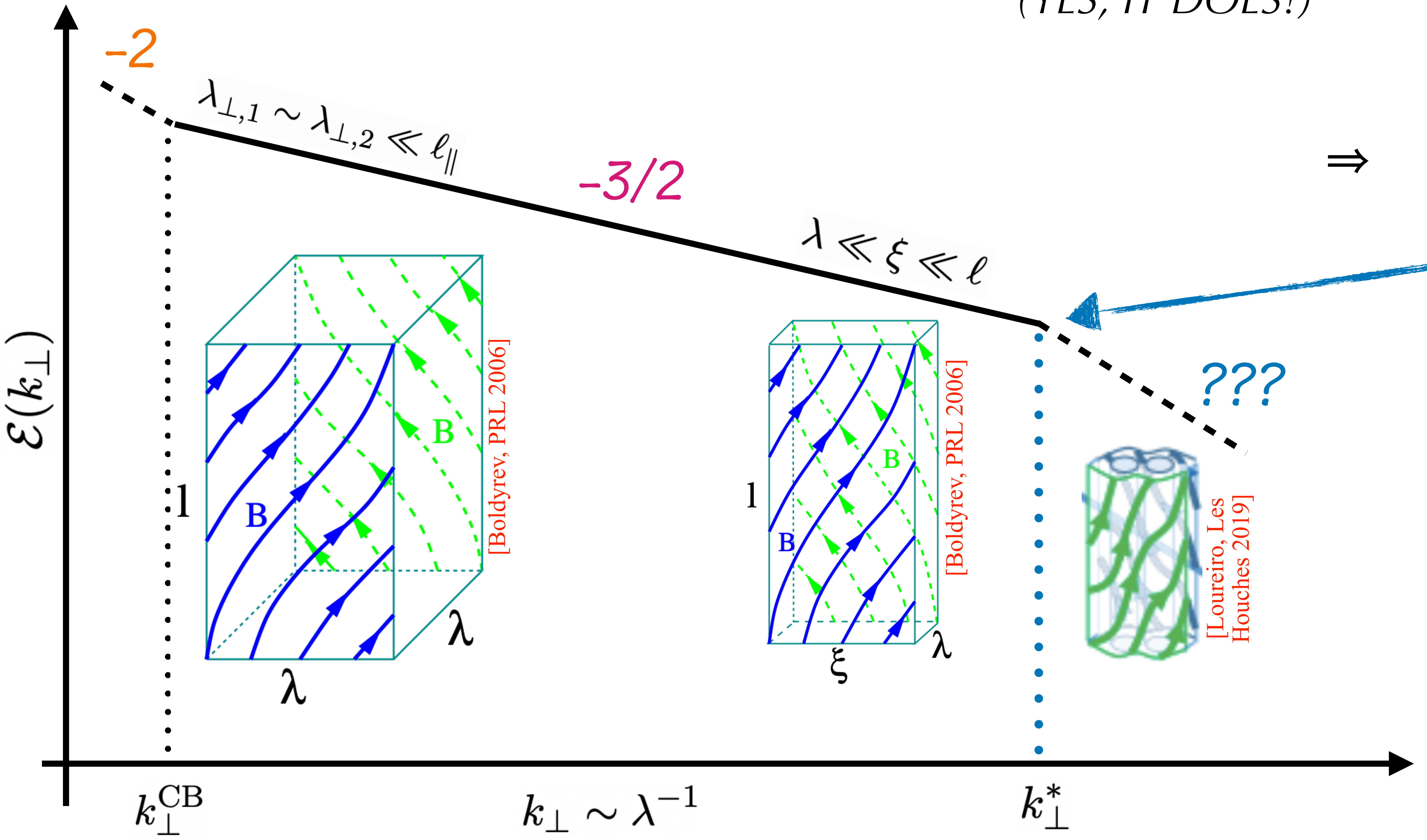

$$\Rightarrow \quad \gamma^{\mathrm{rec}}\,\tau_{\mathrm{nl}} \sim 1 \quad \text{at} \quad \frac{\lambda_*}{\ell_0} \sim S_0^{-4/7}$$

transition to reconnection–mediated cascade

⚠️ this regime exists if the Lundquist number is large enough to separate such transition scale from the actual dissipation scale!

(see references at the beginning of this part)

Recent Developments in Theoretical Models

reconnection-mediated regime in Alfvénic turbulence

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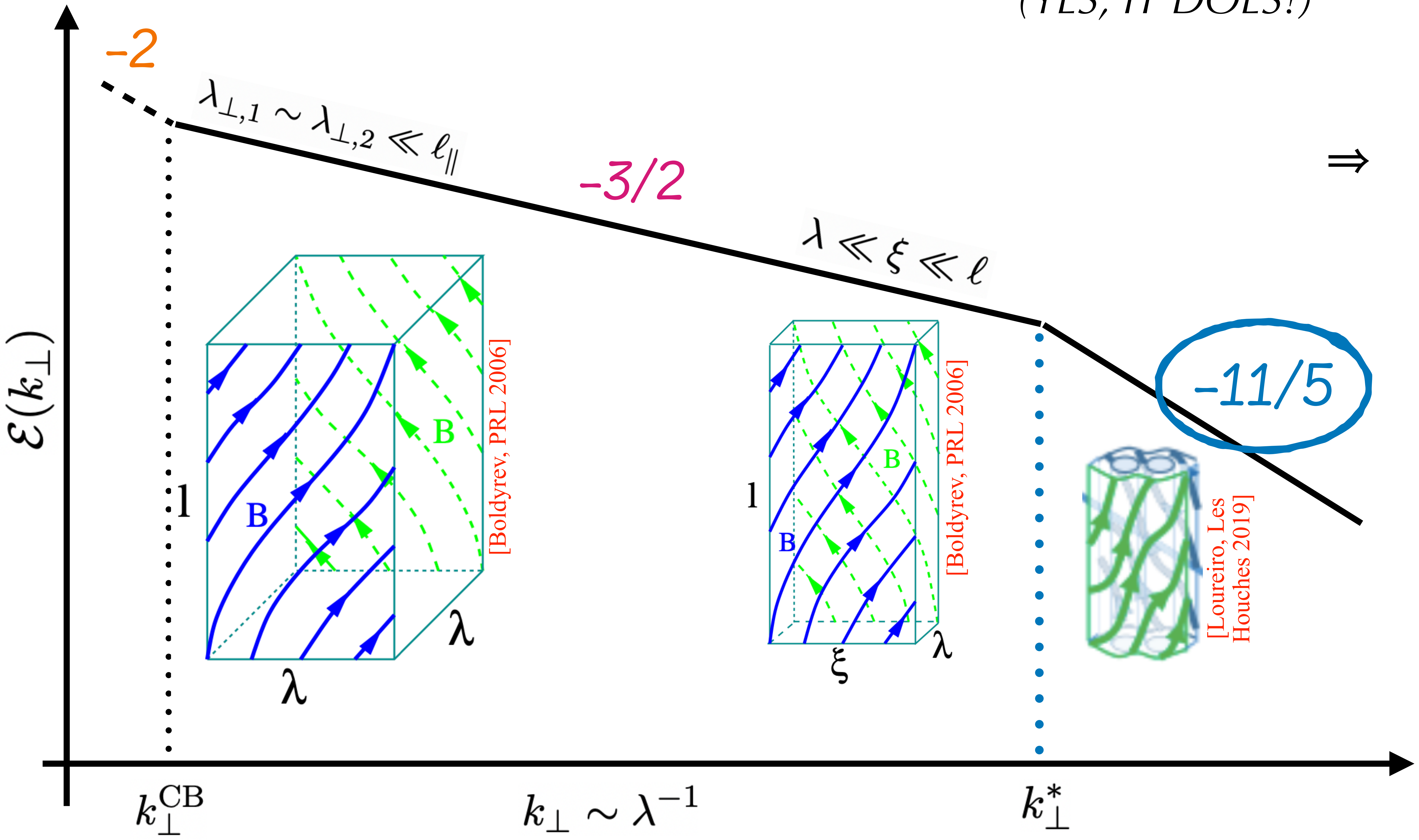


$$\Rightarrow \quad \gamma^{\mathrm{rec}}\,\tau_{\mathrm{nl}} \sim 1 \quad \text{at} \quad \frac{\lambda_*}{\ell_0} \sim S_0^{-4/7}$$

☞ reconnection now defines the cascade time:

$$\tau_{\mathrm{nl}} \longrightarrow \tau_{\mathrm{rec}} \sim 1/\gamma^{\mathrm{rec}}$$

$$\Rightarrow \quad \mathcal{E}(k_\perp) \propto k_\perp^{-11/5}$$

spectrum of reconnection-mediated turbulence

Recent Developments in Theoretical Models

Is this the end of the story?!

Of course not! ... we have totally neglected KINETIC EFFECTS...

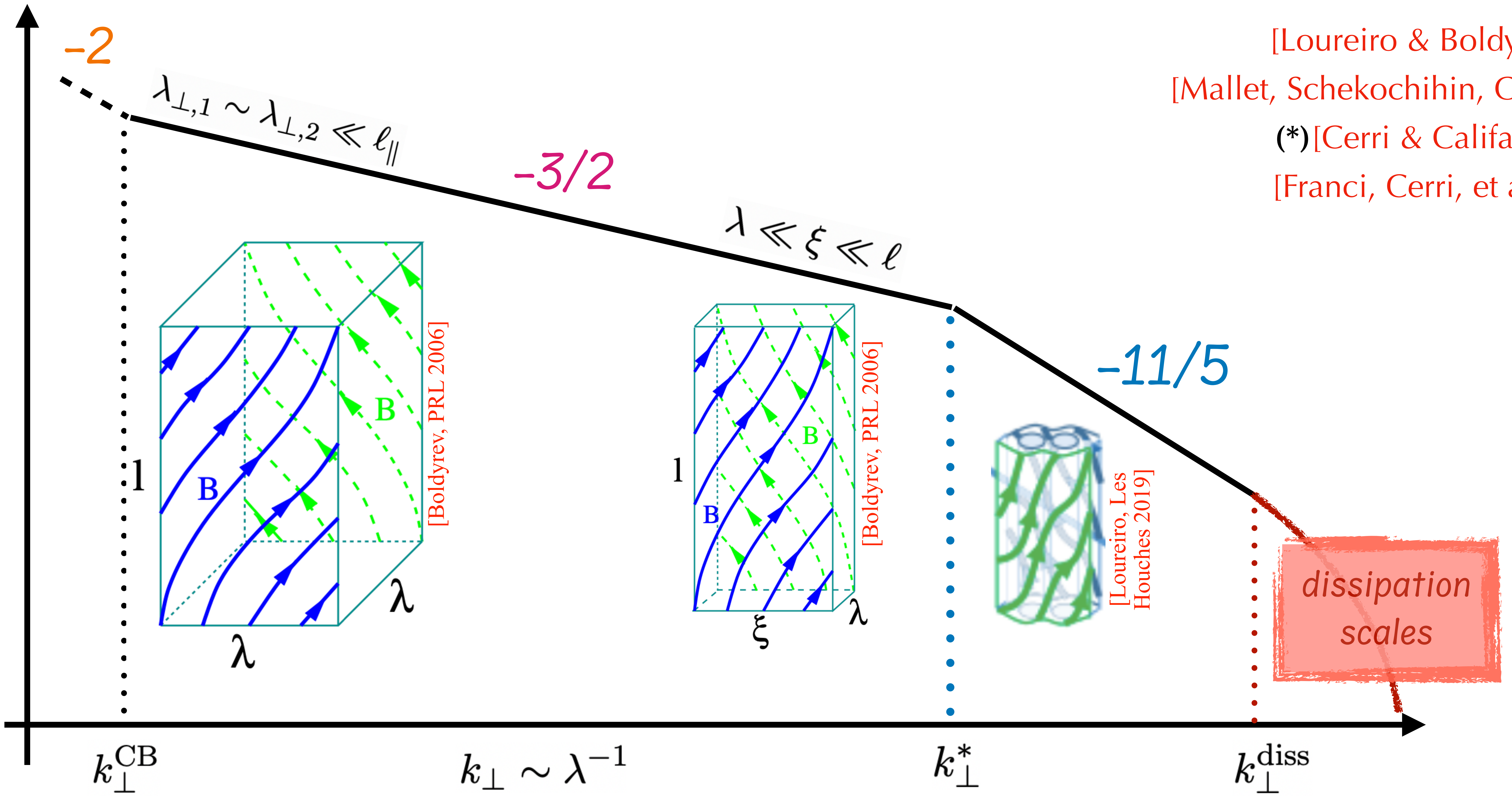
☞ *reconnection-mediated turbulence with kinetic effects (theory & simulations):*

[Loureiro & Boldyrev, ApJ 2017]

[Mallet, Schekochihin, Chandran, JPP 2017]

(*)[Cerri & Califano, NJP 2017]

[Franci, Cerri, et al., ApJL 2017]

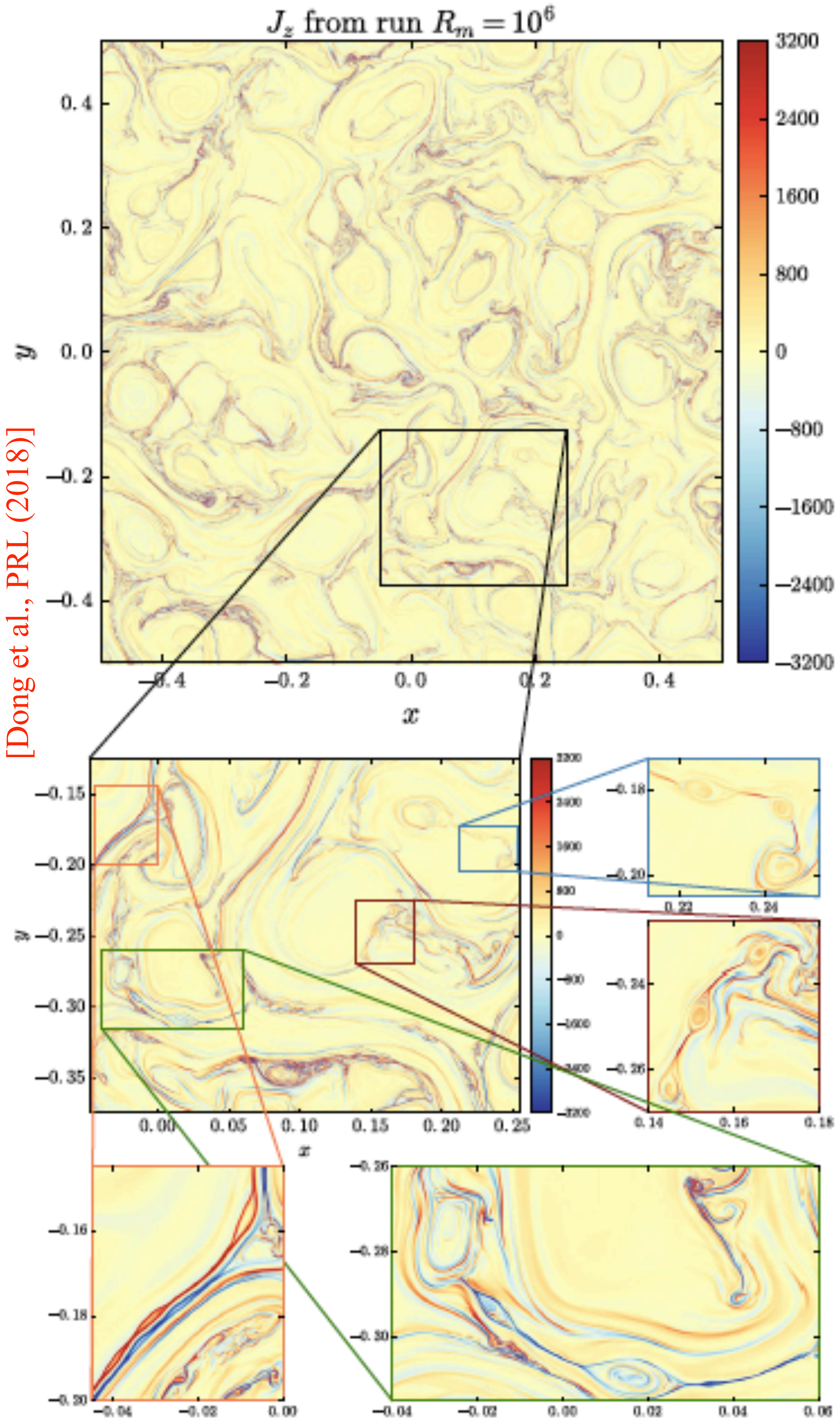


(*) *Actually this was the first suggestion of the existence of a reconnection-mediated regime: from a kinetic simulation, before any theory existed!*

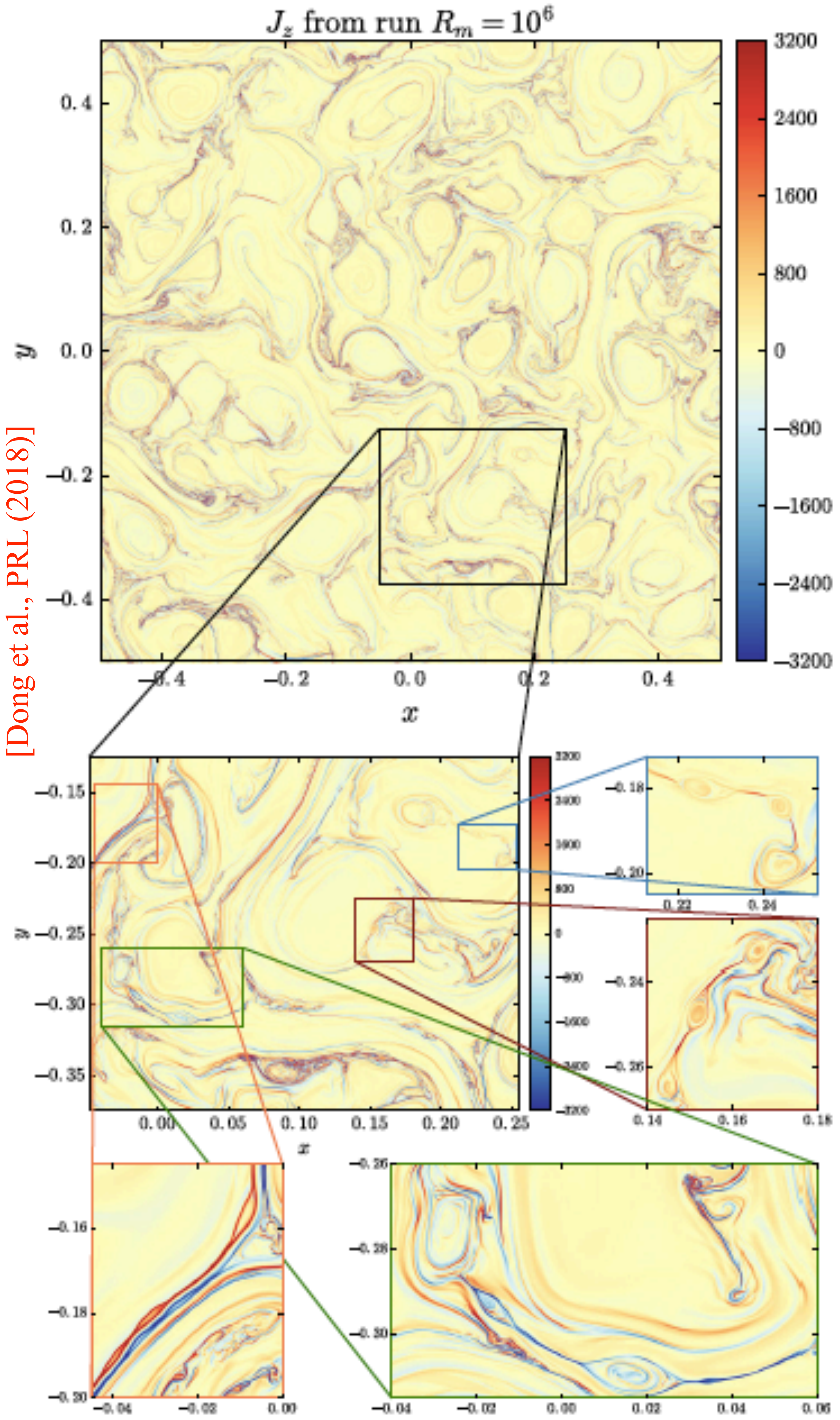
Progress via Numerical Simulations

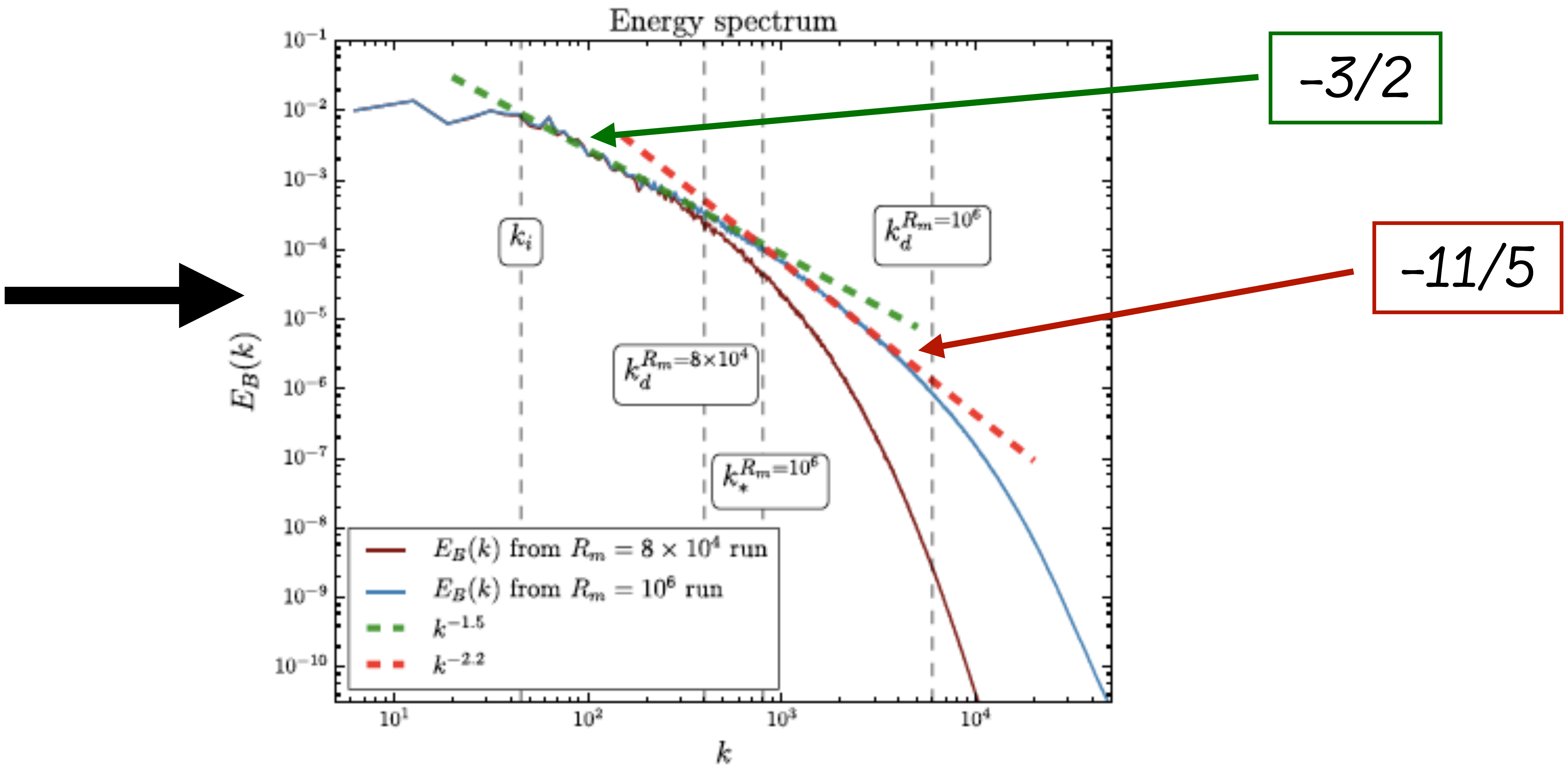
Recent Developments via Numerical Simulations

[Dong et al., PRL (2018)]



Recent Developments via Numerical Simulations





☞ so far, the only evidence of reconnection-mediated turbulence in MHD

- only in **2D geometry**
- requires ***extremely large Lundquist numbers*** (grid: 64000^2 !!!)

Recent Developments via Numerical Simulations

Can we do better, and can it be done in 3D?

Recent Developments via Numerical Simulations

Can we do better, and can it be done in 3D?

YES!

just go back to a basic 3D setup: start from the ***building blocks of the Alfvénic cascade***!

Recent Developments via Numerical Simulations

[Cerri, Passot, Laveder, Sulem, Kunz, ApJL (*to be submitted*)]

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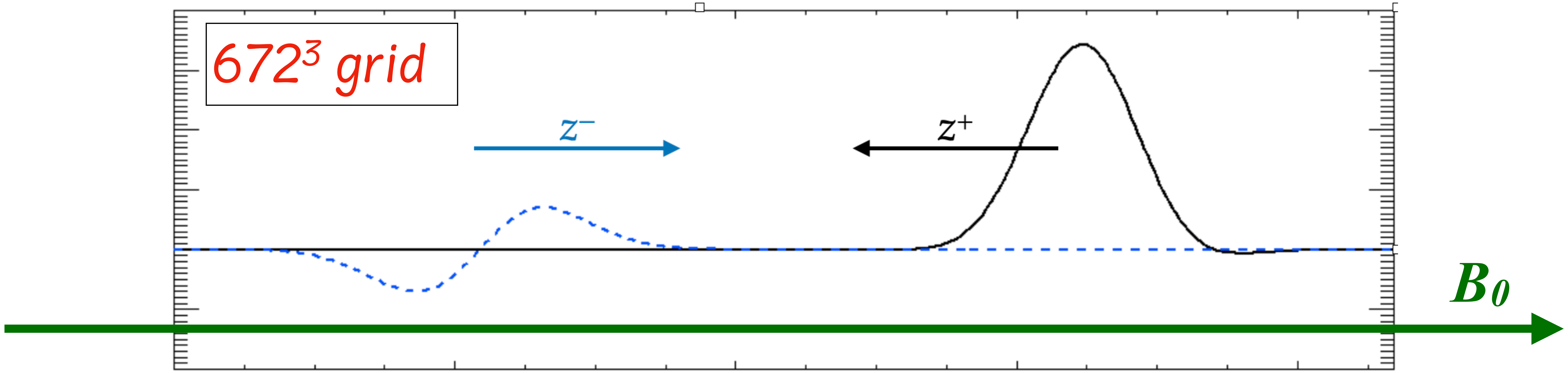

Simulations performed with the ***Hamiltonian 2-fields gyro-fluid*** model/code by Passot, Tassi, Sulem, and Laveder

☞ *model retains* ***only Alfvén & kinetic-Alfvén modes****, assumes* ***strong anisotropy*** ($k_{\|} << k_{\perp}$), …

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But we do it "WISELY", i.e., with a "trick":

$$\gamma^{\rm rec}\,\tau_{\rm nl} \sim 1$$

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Usually, one tries to increase $\gamma^{\rm rec}$ by achieving large S: requires extreme resolution!

Recent Developments via Numerical Simulations

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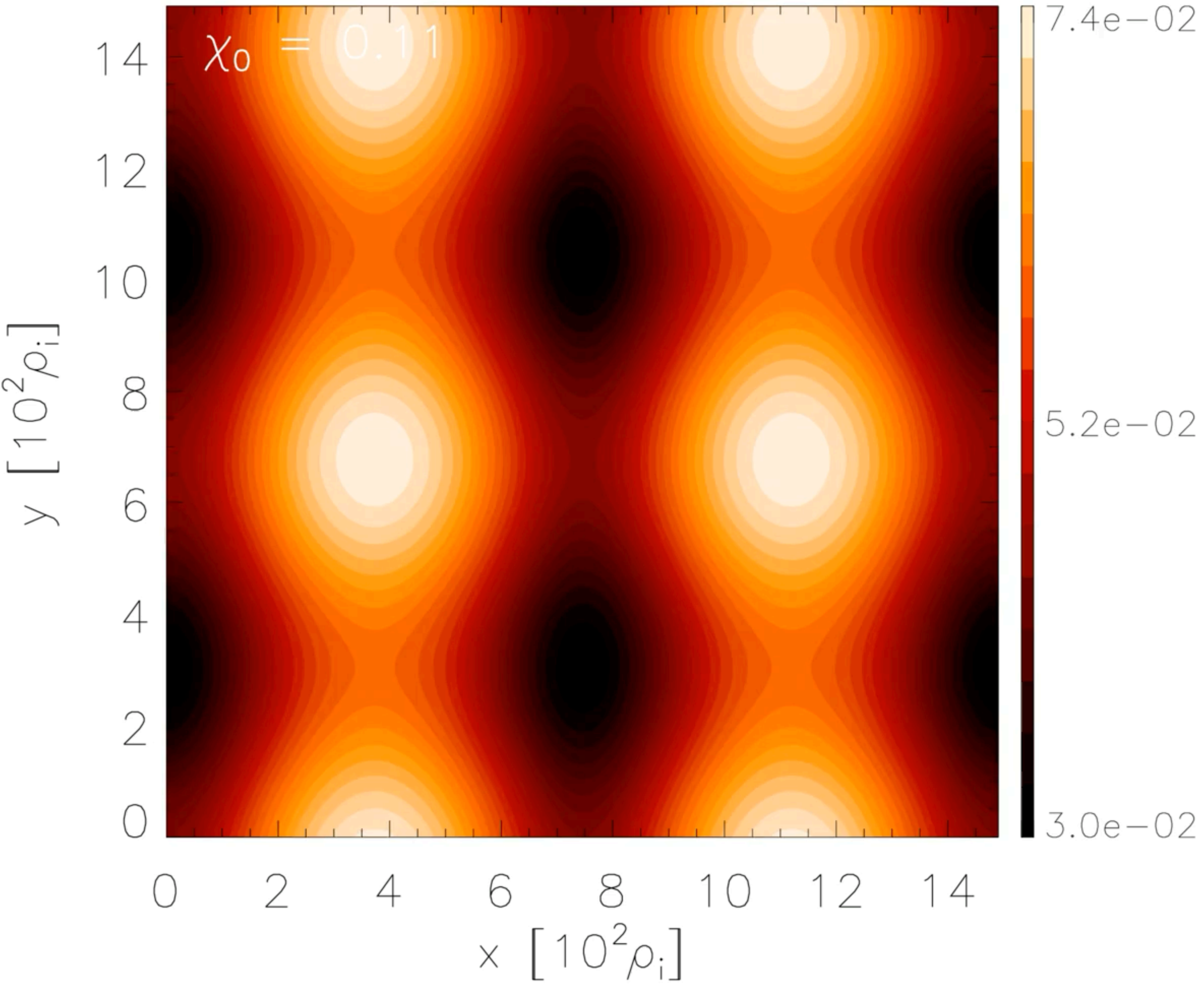
Let's increase the non-linear time instead! (by considering a smaller non-linear parameter, $\chi < 1$)

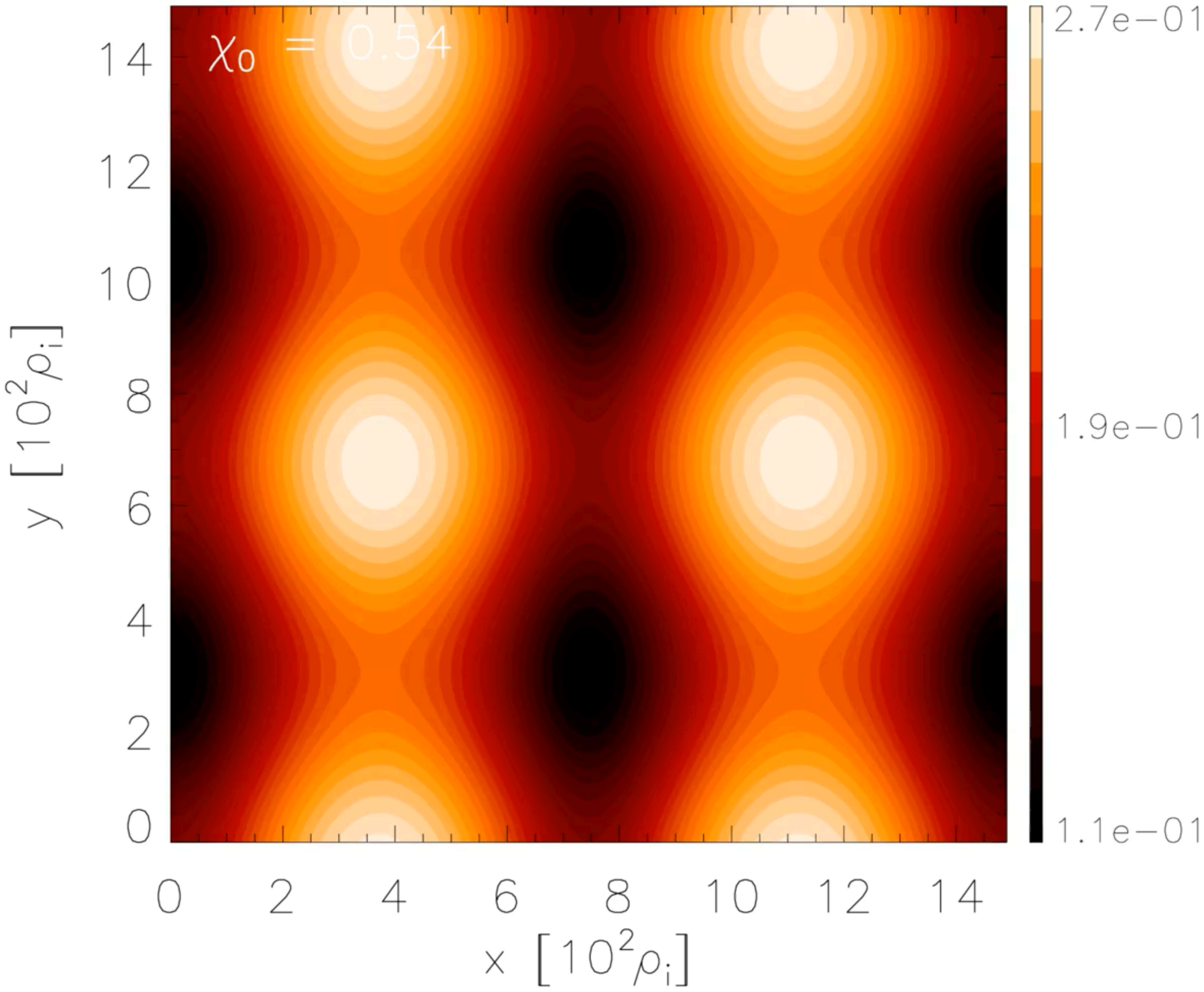
Recent Developments via Numerical Simulations

[Cerri, Passot, Laveder, Sulem, Kunz, ApJL (*to be submitted*)]

$<\delta \mathbf{b}_\perp>_z / \mathbf{B}_0 \ (\chi_0 \sim 0.1)$



$<\delta \mathbf{b}_\perp>_z / \mathbf{B}_0 \ (\chi_0 \sim 0.5)$



Recent Developments via Numerical Simulations

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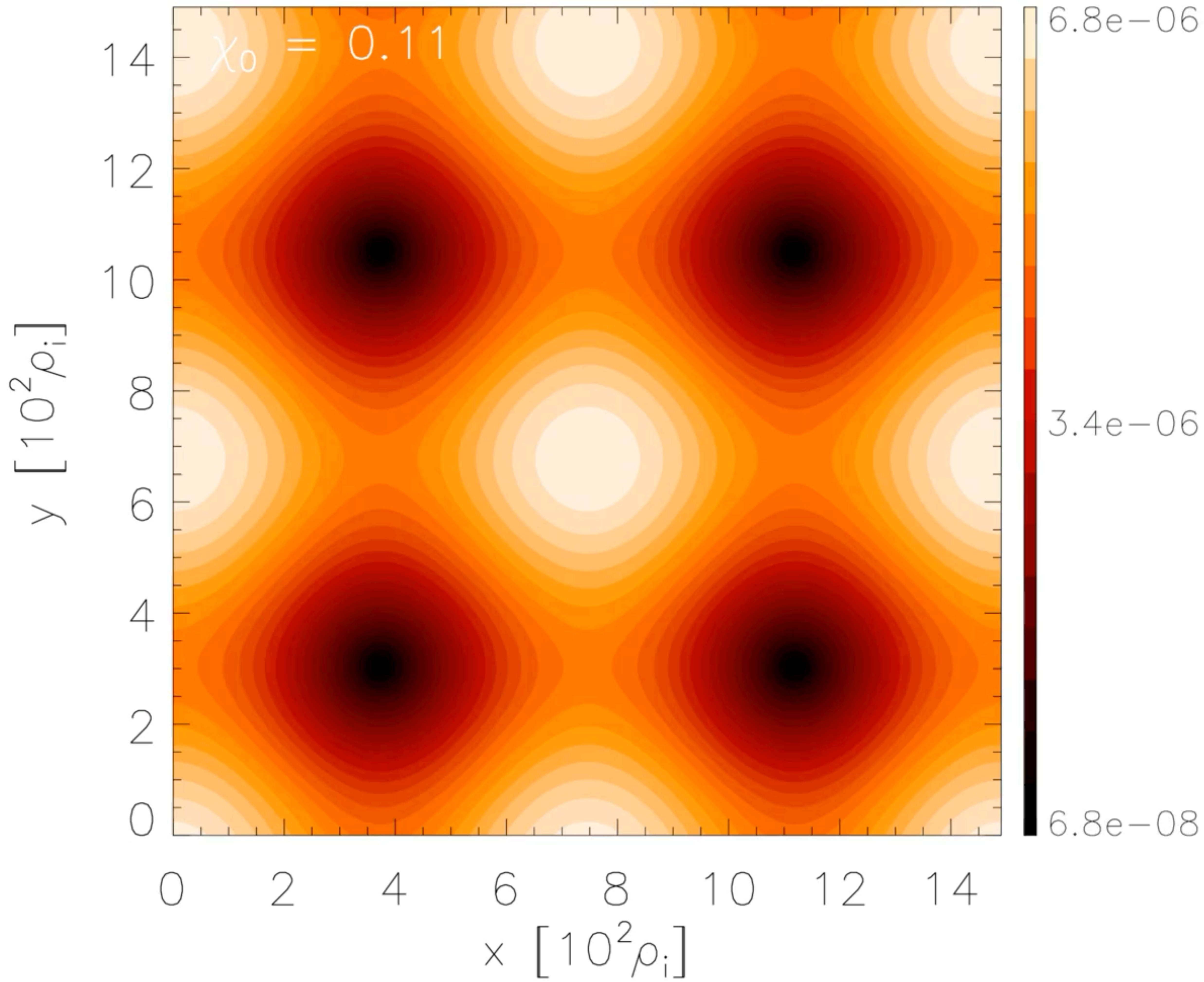
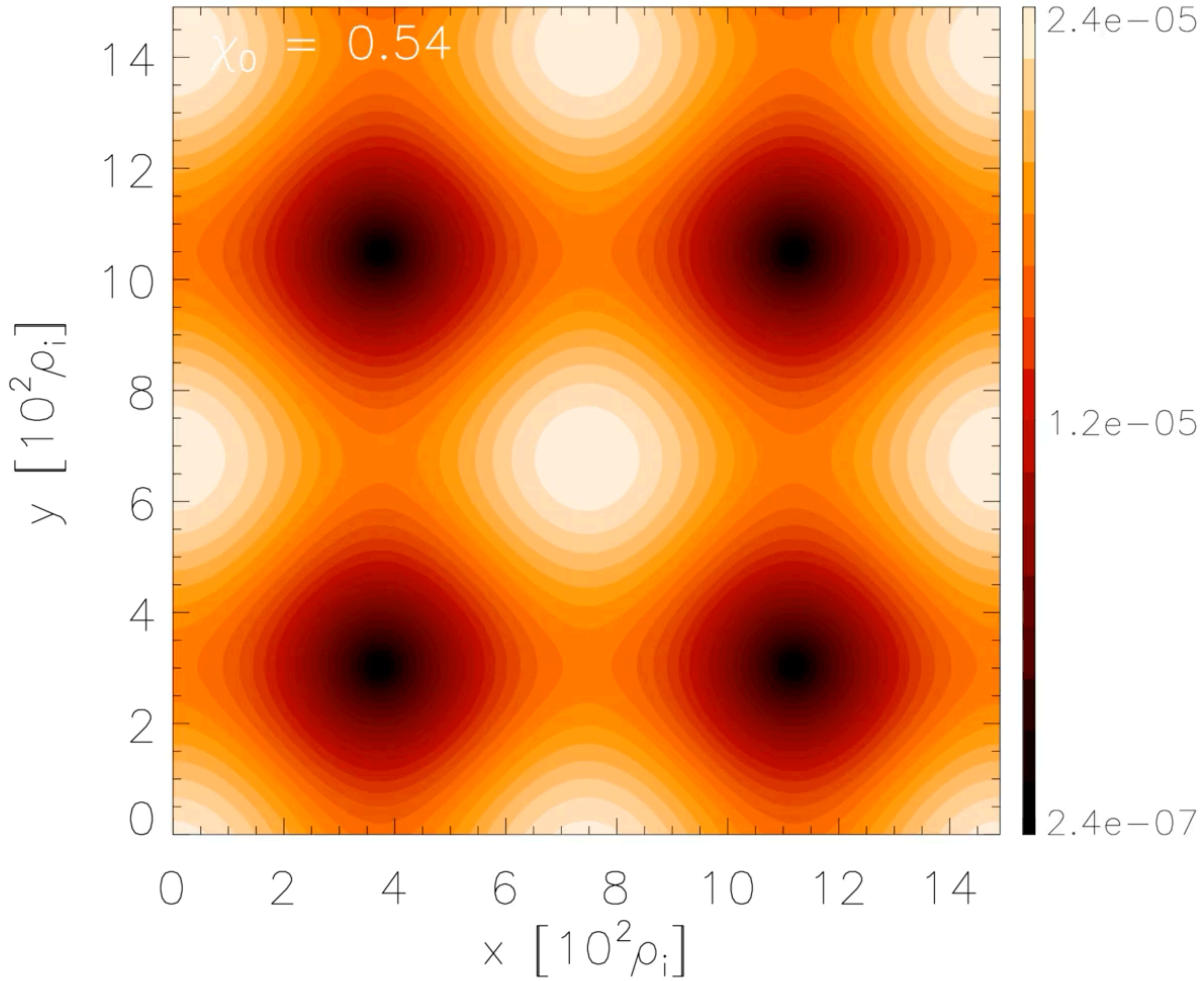
$\delta \mathbf{b}_\perp |_{z=L/2} / \mathbf{B}_0 \ (\chi_0 \sim 0.1)$



$\delta \mathbf{b}_\perp |_{z=L/2} / \mathbf{B}_0 \ (\chi_0 \sim 0.5)$



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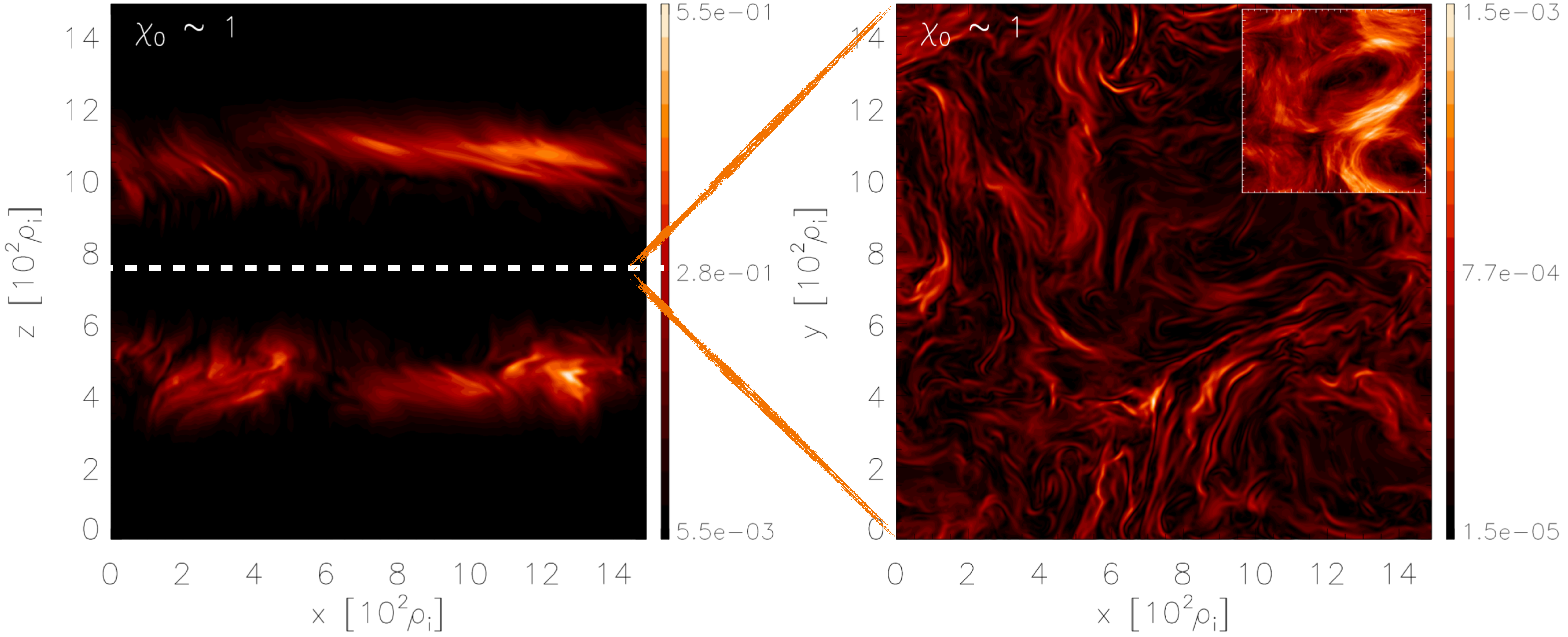

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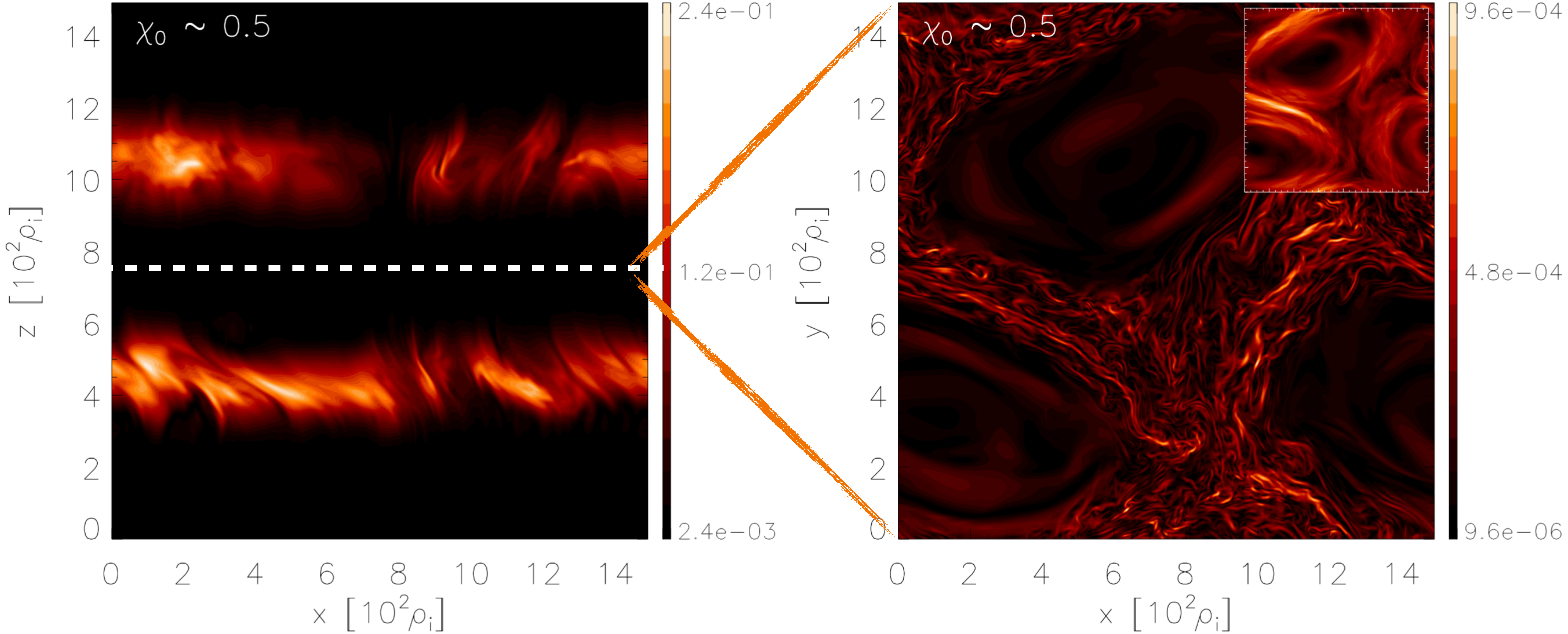
$\delta b_\perp / \mathbf{B}_0$ ($\chi_0 \sim 0.5$)



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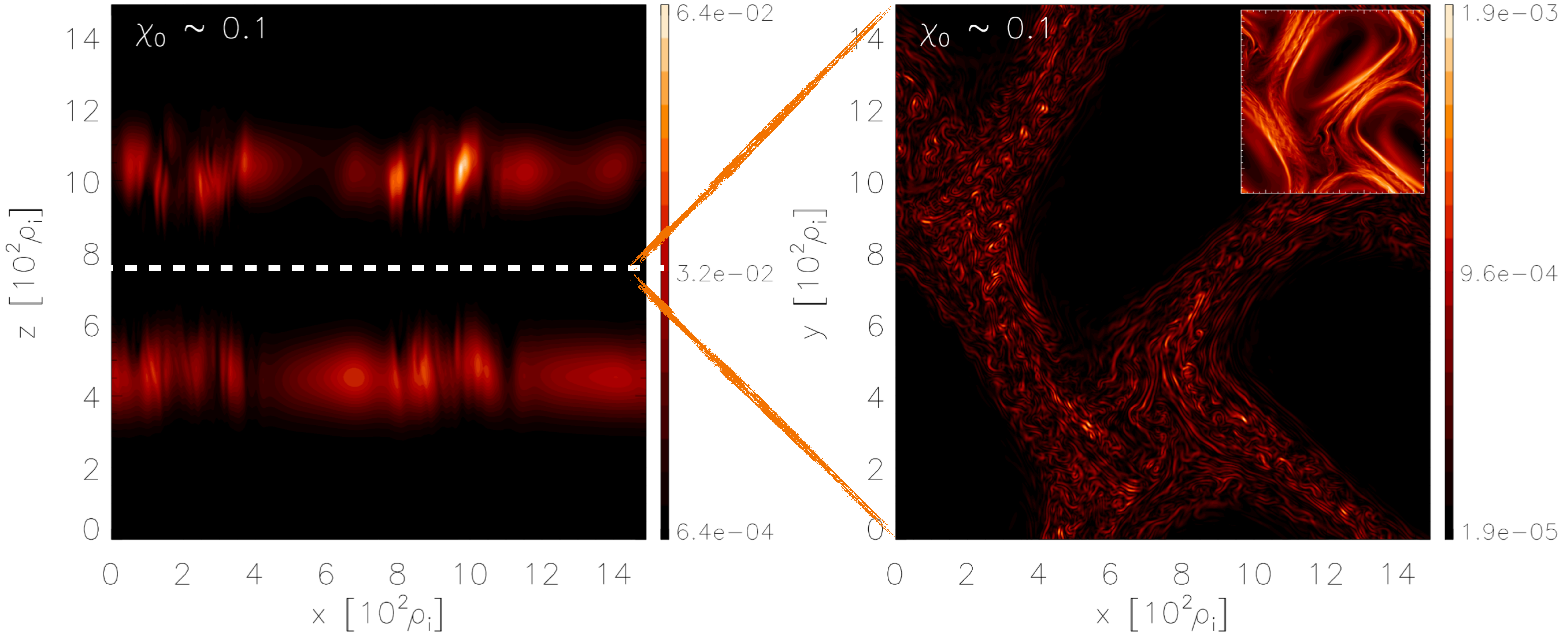

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[Cerri, Passot, Laveder, Sulem, Kunz, ApJL (*to be submitted*)]

Take home message(s)

☞ ***The fate of weak MHD turbulence is to become strong…***

…but which type of strong MHD turbulence?

☞ ***A new turbulence regime exists, mediated by magnetic reconnection***

…emergence of reconnection-mediated turbulence depends both on the Lundquist number and/or on the strength of the nonlinearities

☞ We have now provided a ***proof via 3D simulations (from a first-principle setup)***

☞ **BUT: there are still a lot of open questions… we need smart(*er than me*) people!**

Thank you for your attention!