From phase turbulence to solitons in semiconductor laser frequency combs

M. Piccardo*

Key collaborators

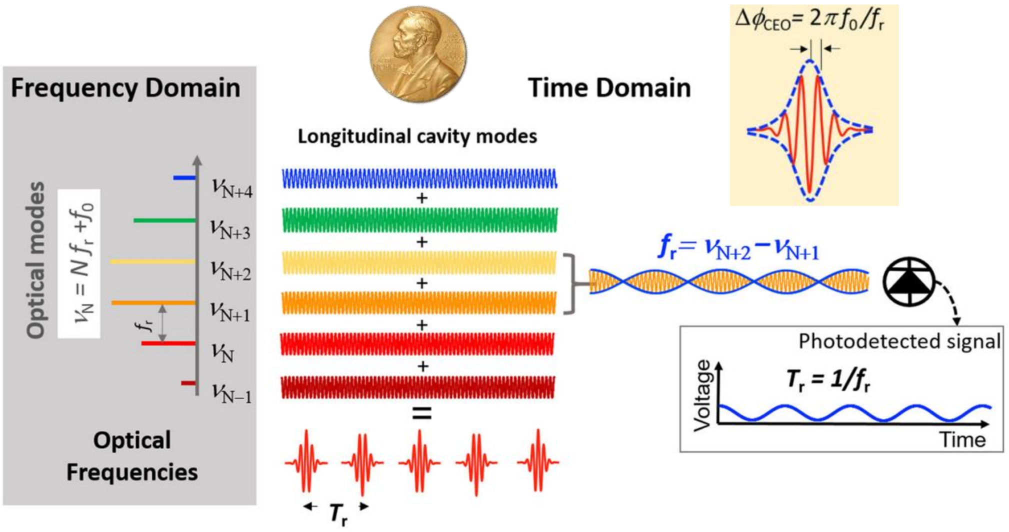
Harvard: D. Kazakov, F. Capasso
TU Wien: N. Opacak, M. Beiser, J. Hillbrand, B. Schwarz
U Bari, U Insubria, PoliMi: L. Columbo, F. Prati, M. Brambilla, A. Gatti, L. A. Lugiato
PoliTo: C. Silvestri, M. Gioannini

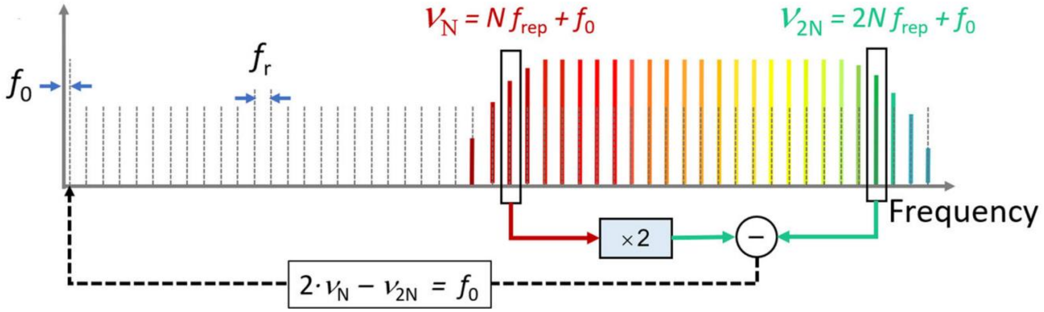
Harvard John A. Paulson **School of Engineering** and Applied Sciences
TU WIEN
ISTITUTO ITALIANO DI TECNOLOGIA

*piccardo@g.harvard.edu

Optical frequency combs

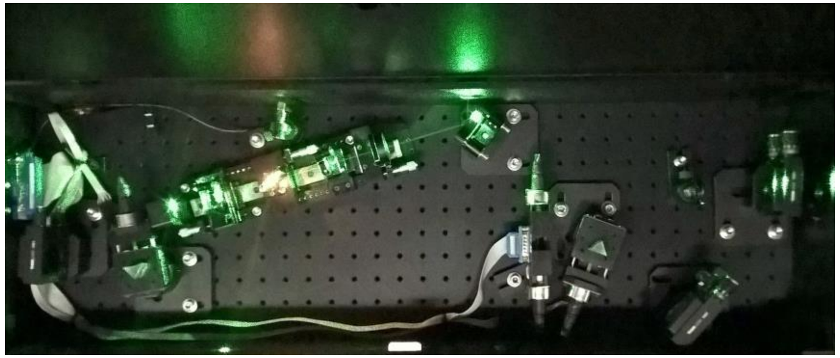
“Rulers” in the frequency domain



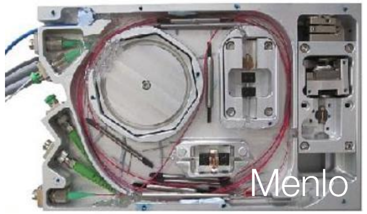


- Perfectly equidistant optical lines
- Low phase and amplitude noise

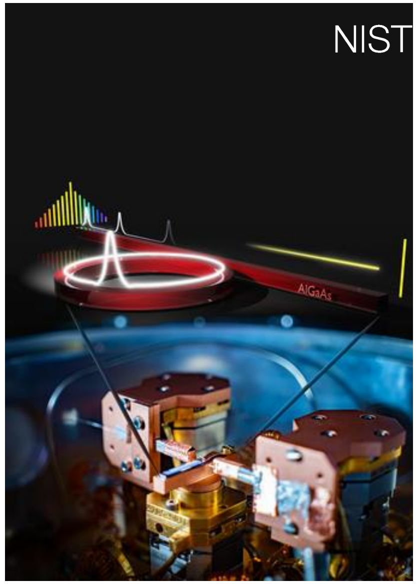
Ti:sapphire fs laser



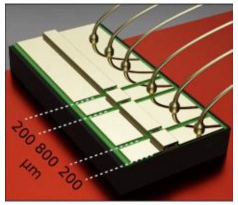
Fiber systems



Kerr microresonators

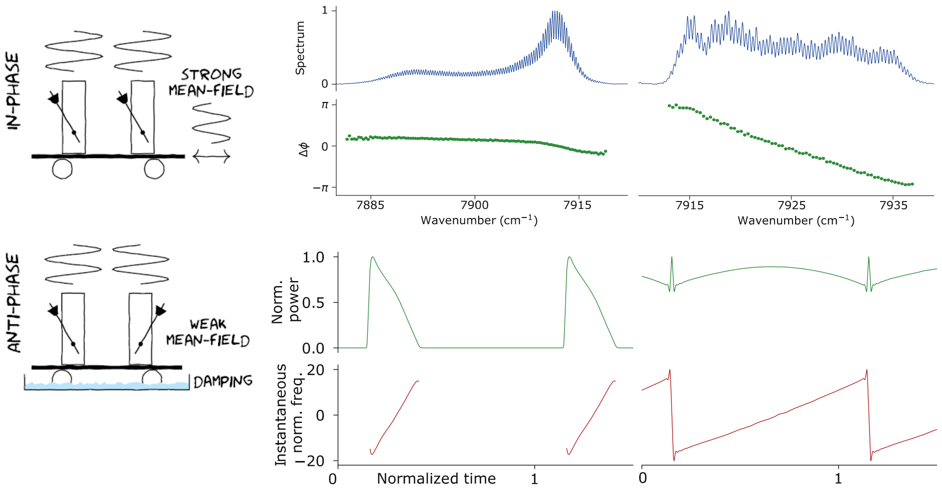


Semiconductor lasers



AM combs

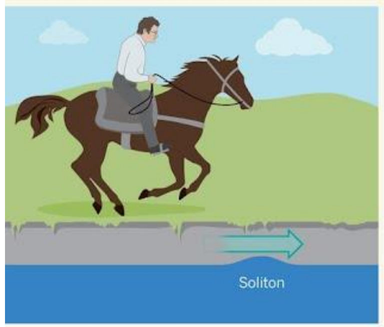
FM combs



Many kinds of temporal waveforms

Cavity solitons

Solitary wave in water



Scott Russell, 1834

Key properties:

- Rigid form
- Localized within a region
- Upon interaction with other solitons remain unchanged (minus a phase shift)

Cavity soliton:
Dissipative soliton circulating in a driven nonlinear cavity

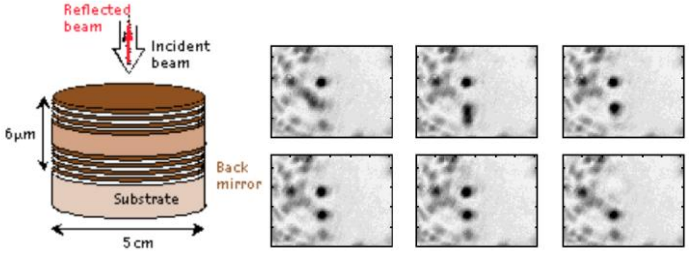
Quantum cascade lasers?

2D: Spatial CS

1D: Temporal CS

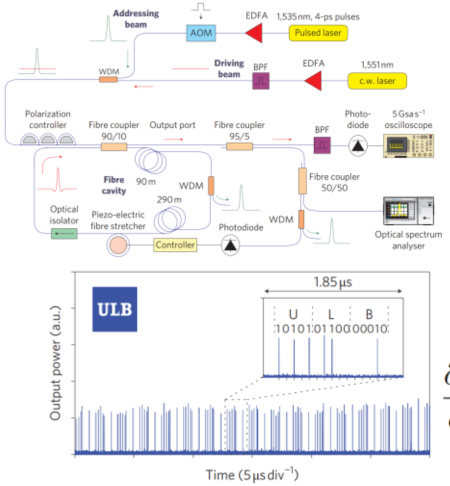
3D: Cavity light bullets

Semiconductor microcavities

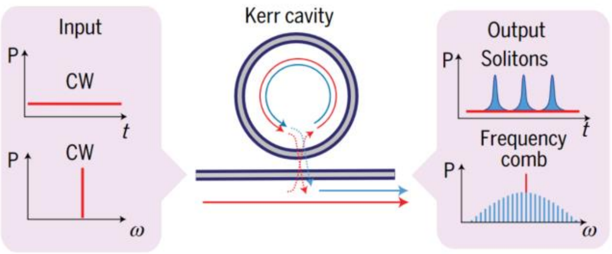


Write and erase solitons

Fibers



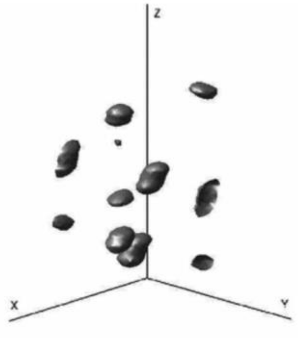
Kerr microresonators



Lugiato-Lefever equation

$$\frac{\partial A}{\partial t} - i\frac{D_2}{2}\frac{\partial^2 A}{\partial \phi^2} - ig|A|^2 A + (\kappa/2 + i\Delta)\, A = \sqrt{\eta\kappa s}$$

Nonlinear resonator with saturable absorber



Transverse and longitudinal confinement

S. Barland *et al.*, Nature 419, 699 (2000)
W. J. Firth and C. O. Weiss, OPN 13, 54 (2002)

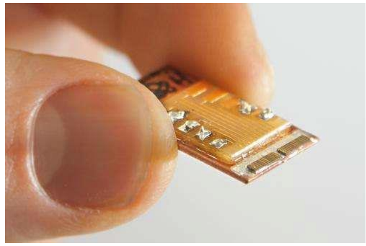
F. Leo *et al.*, Nat. Photon. 4, 471 (2010)

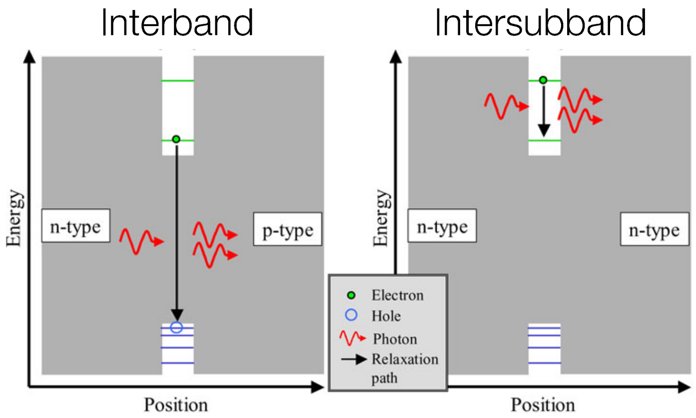
T. J. Kippenberg *et al.*, Science 361, eaan8083 (2018)

M. Brambilla *et al.*, PRL 93, 203901(2004)

What makes quantum cascade lasers unique?

- Unipolar semiconductor lasers
- Emission can be tailored from the mid-IR to the THz by bandstructure engineering
- CW and pulsed mode up to several watts of optical power



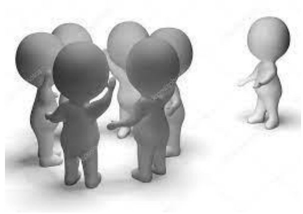


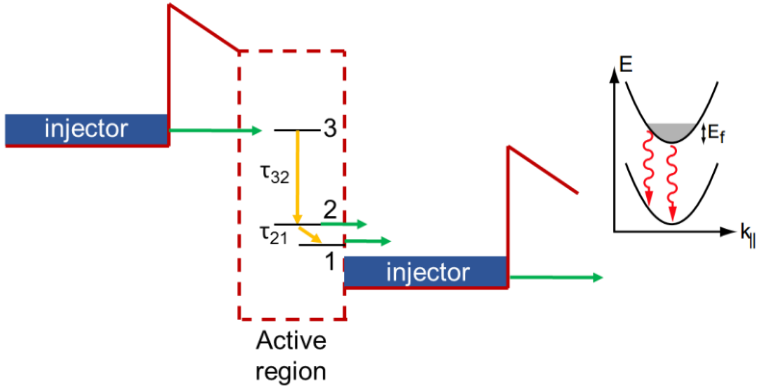
$T_1 \cong 1$ ns

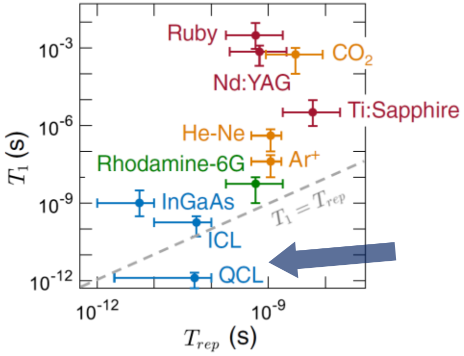
Bipolar s.c. lasers

$T_1 \cong 1 - 10$ ps

QCLs







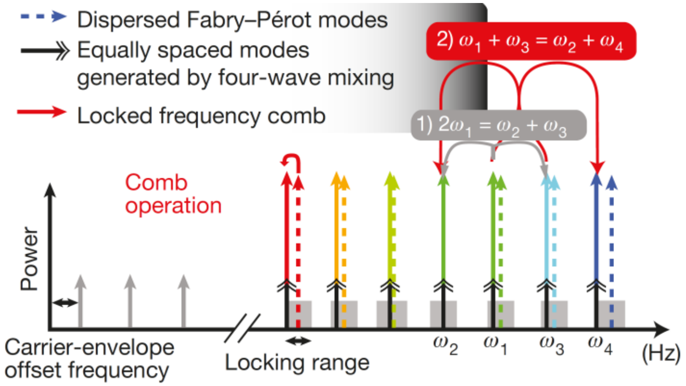
- Fast gain recovery
- Class-A lasers: the electric field is the slowest variable
- Low but non-zero value (0.2÷3) of the linewidth enhancement factor

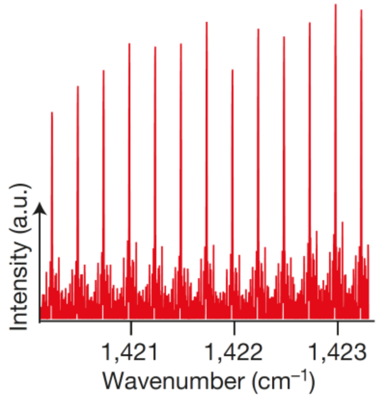
$$\Delta\nu = (1 + \alpha^2)\Delta\nu_{ST}$$

J. Faist, *Quantum Cascade Lasers,* Oxford University Press (2013) M. Piccardo et al., PRL 122, 253901 (2019) M. Piccardo and F. Capasso, Laser & Photonics Rev. 16, 2100403 (2022)

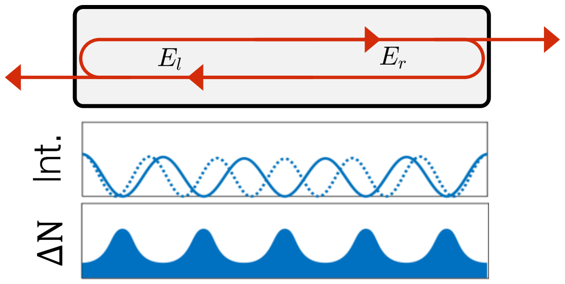
Fabry-Perot QCL frequency combs

First demonstrated in 2012, much progress in terms of science and technology since then





Fabry-Perot cavity



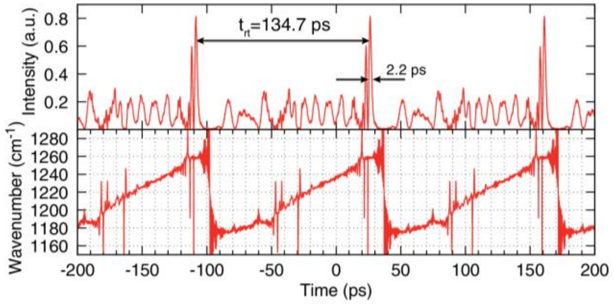
Cavity position

Mechanism

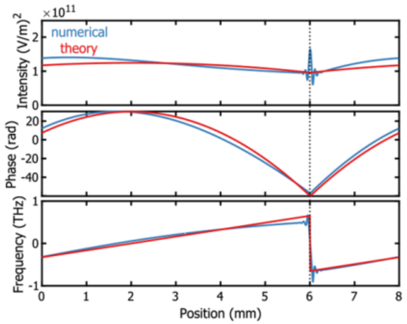
- Counter-propagating waves → Spatial hole burning couples modes
- Incoherent instability → Multimode operation
- Four-wave-mixing assures phase-locking

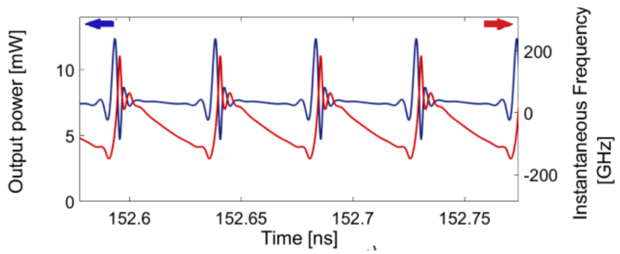
(Quasi-) Frequency modulated combs

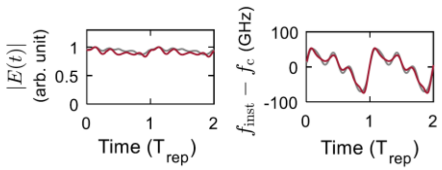
Experiments

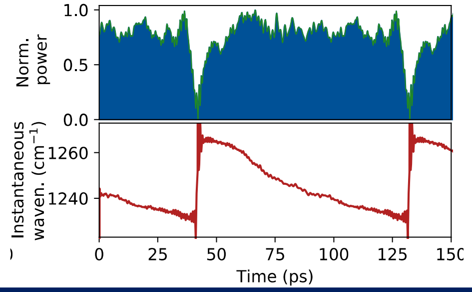


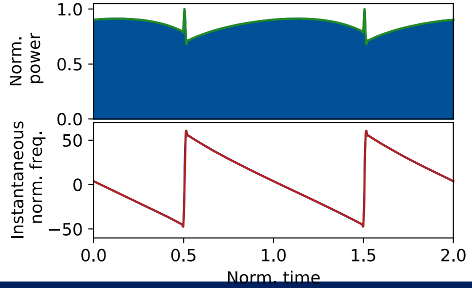
Theory





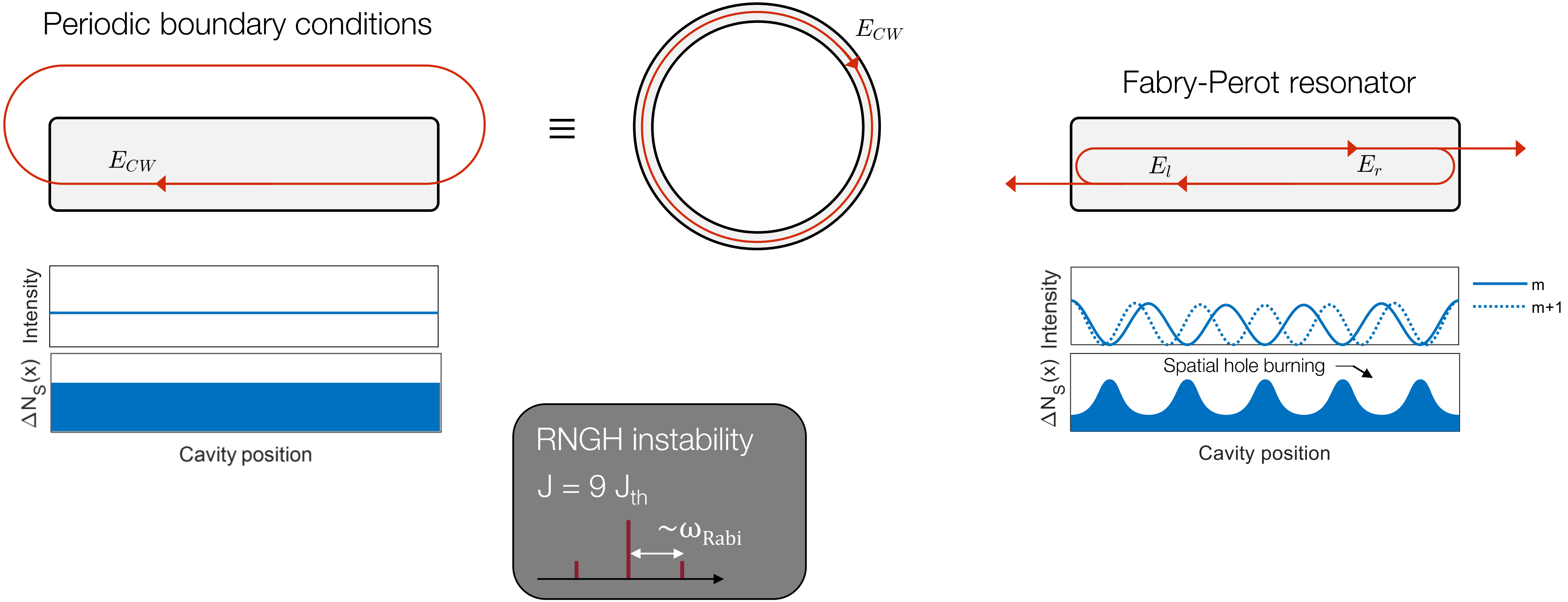






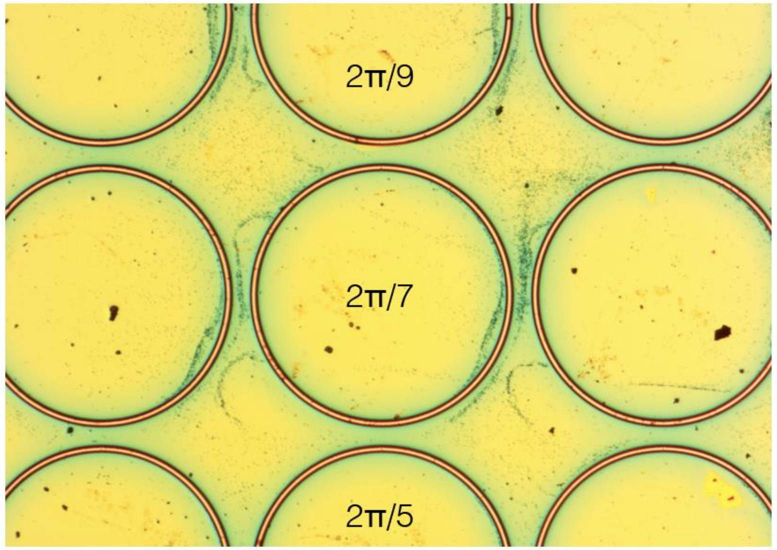
A. Hugi *et al.*, Nature 492, 229 (2012)
M. Singleton *et al.*, Optica 5, 948 (2018)
M. Piccardo et al., PRL 122, 253901 (2019)
D. Burghoff, Optica 7, 1781 (2020)
N. Opacak *et al.*, Phys. Rev. Lett. 123, 243902 (2019)
C. Silvestri *et al.*, Opt. Express. 28, 23846 (2020)

A multimode instability in a ring QCL is not expected!

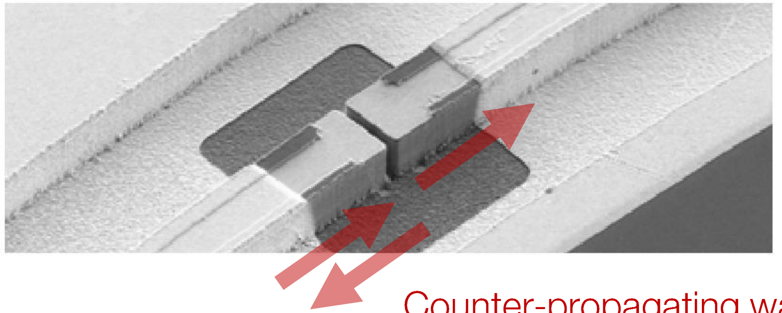


This would be an extremely high injection level, extremely impractical experimentally!

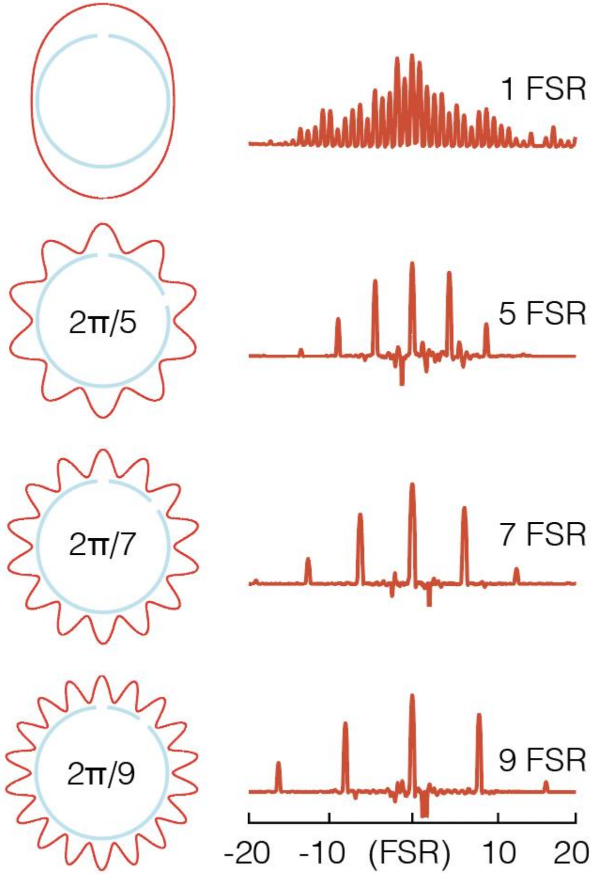
Inducing an instability in ring QCLs by defect engineering



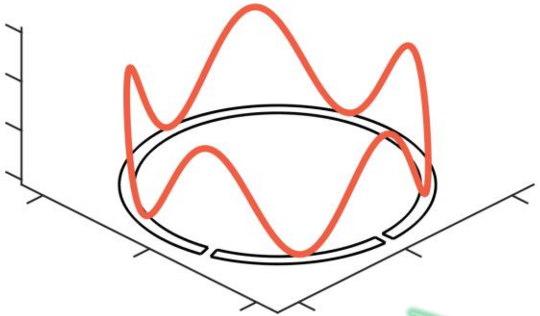
Engineered defect

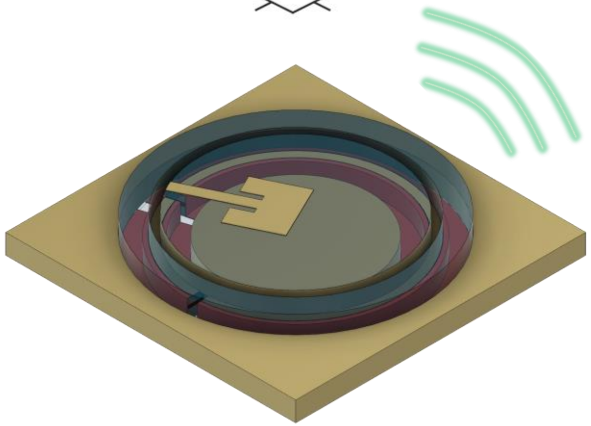


Counter-propagating waves



Sub-THz laser radio transmitters

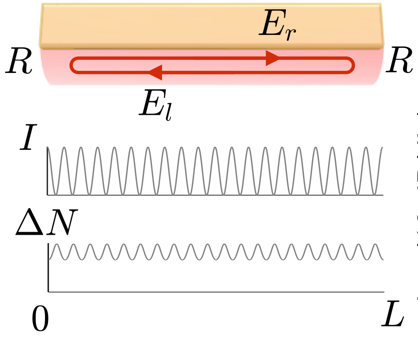


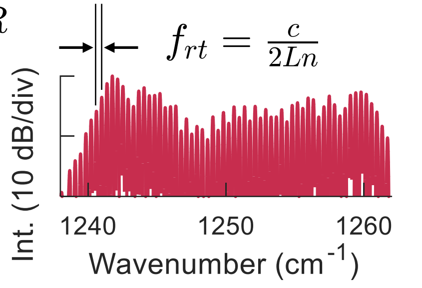


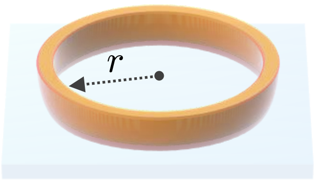
D. Kazakov *et al.*, Optica, 8, 1277 (2021)

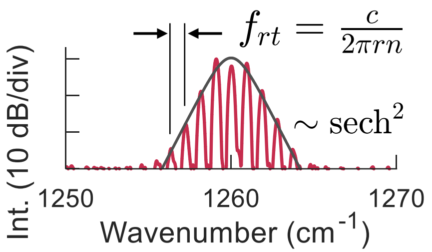
M. Piccardo *et al.*, PNAS 116, 9181 (2019)

Instability in ring QCLs without defects



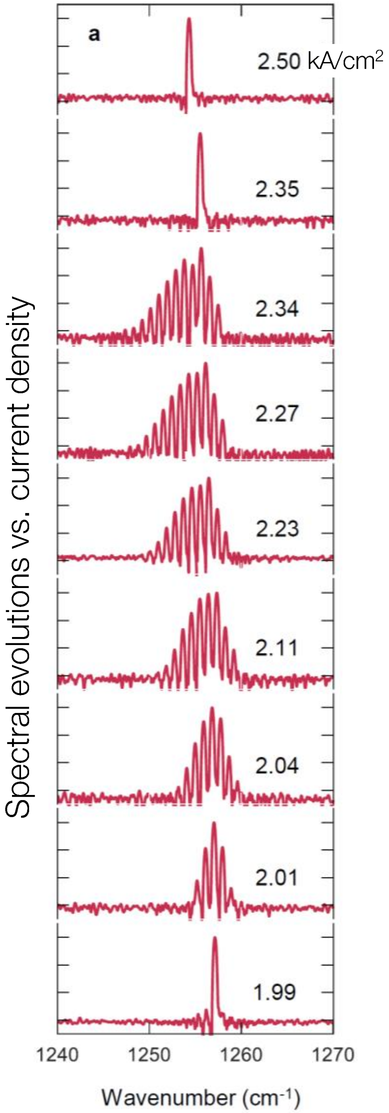


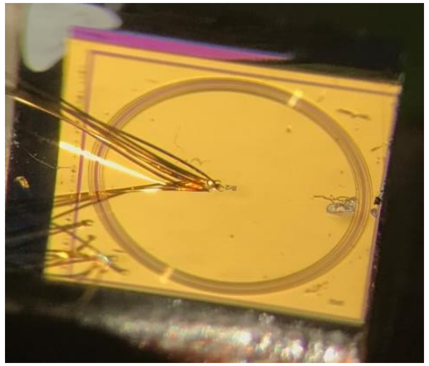




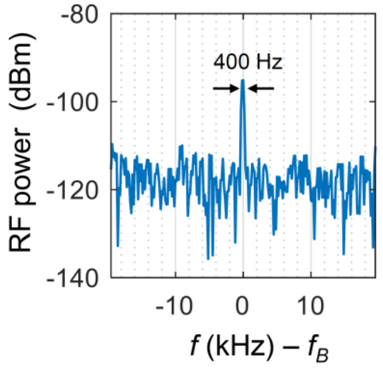
No spatial hole burning:

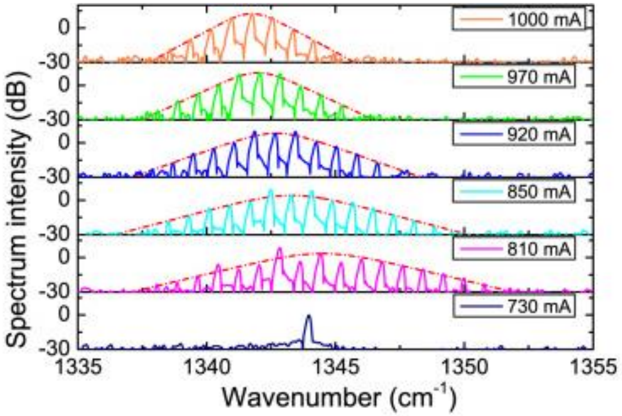
Needs an explanation!



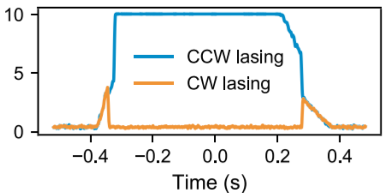


Narrow RF beat note





Spontaneous symmetry breaking



Minimal amount of optical backscattering favors competition between directional and bidirectional

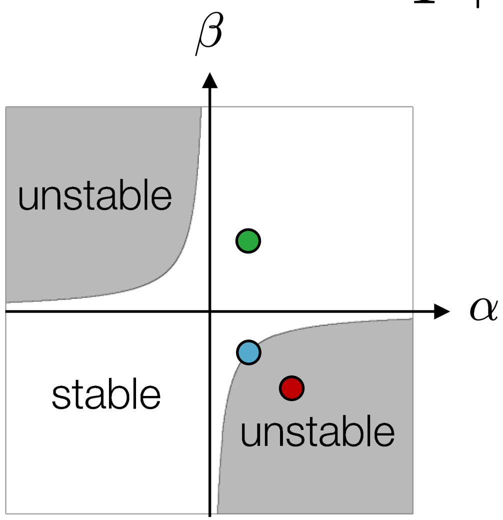
Directional regime wins as self-gain saturation is smaller than cross-gain saturation

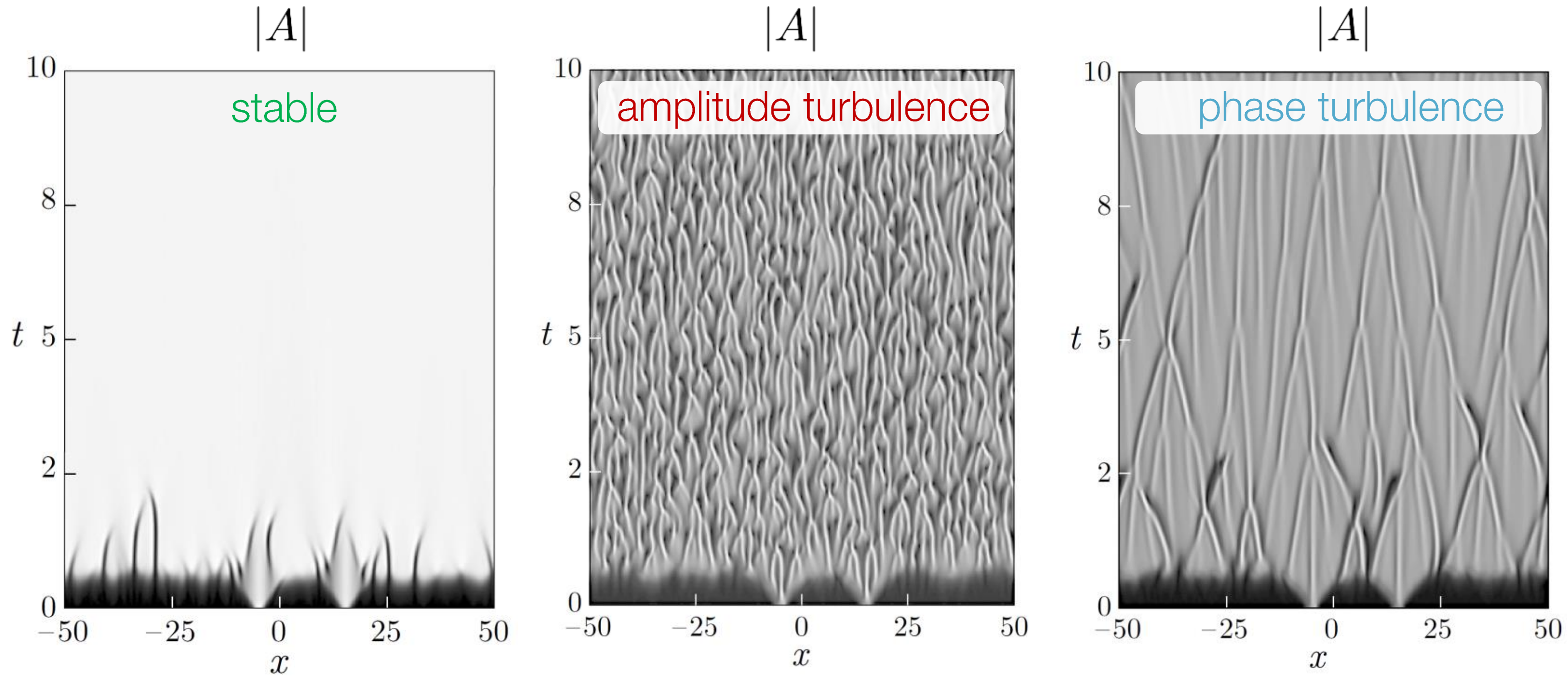
The complex Ginzburg-Landau equation

$$A_t = A - (1 + i\beta)|A|^2 A + (1 + i\alpha) A_{xx}$$

Benjamin-Feir-Newell stability criterion

$$1 + \alpha\beta > 0$$





From the QCL master equation to the CGLE

$$
\begin{aligned}
\frac{n}{c}\partial_t E_\pm \pm \partial_x E_\pm &= i\frac{k''}{2}\partial_t^2 E_\pm + i\beta(|E_+|^2 + |E_-|^2)E_\pm \\
&+ \frac{g_0}{2\left(1+\frac{|E_+|^2+|E_-|^2}{E_{sat}^2}\right)}\frac{1+i\alpha}{1+i\xi}\Bigg\{E_\pm - \frac{T_2}{1+i\xi}\partial_t E_\pm \\
&+ \left(\frac{T_2}{1+i\xi}\right)^2\partial_t^2 E_\pm - \frac{T_g}{T_1 E_{sat}^2}\Bigg[E_\pm|E_\mp|^2 \\
&- \partial_t E_\pm |E_\mp|^2\left(T_g + \frac{T_2}{1+i\xi}\right) - E_\pm E_\mp \partial_t E_\mp^*\left(T_g + \right. \\
&\left. + \frac{T_2}{1+i\xi}\right) - E_\pm E_\mp^* \partial_t E_\mp \frac{T_2}{1+i\xi}\Bigg]\Bigg\} - \frac{\alpha_w}{2}E_\pm
\end{aligned}
$$

- Fast gain medium
- Unidirectional field

CGLE

$$
\partial_t E = E + (1+ic_1)\partial_x^2 E - (1+ic_2)|E|^2 E
$$

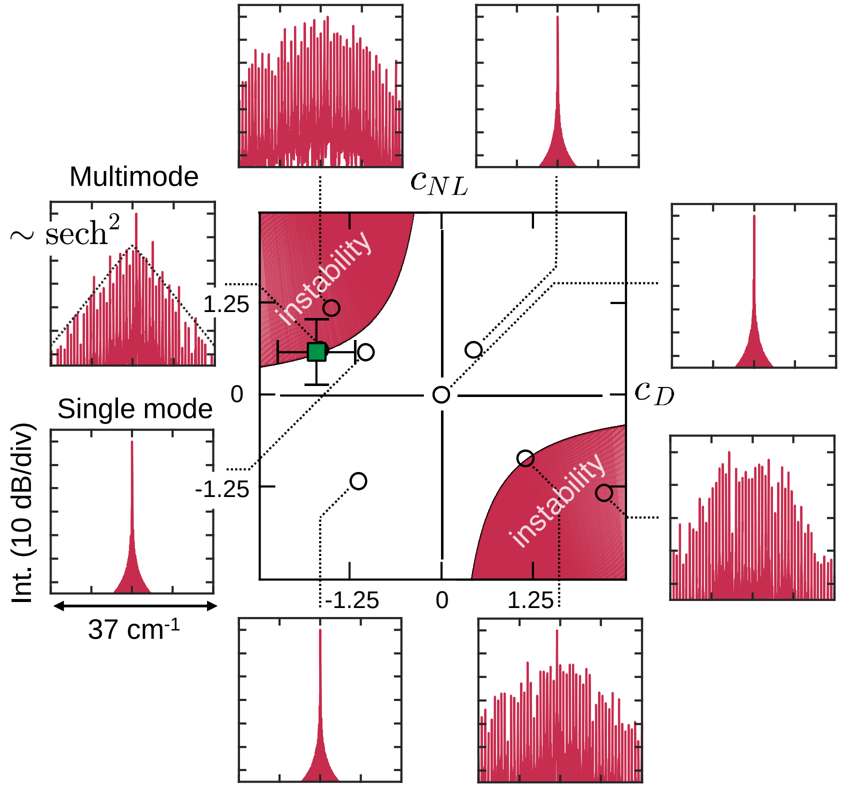
$$
c_1 = \frac{d_I}{d_R} \approx -\frac{\alpha}{2} + \frac{g_0}{\alpha_w(2g_0-\alpha_w)T_2^2}k''
$$

$$
c_2 = \frac{n_I}{n_R} \approx \frac{\alpha}{2} - \frac{2P_{sat}g_0}{\alpha_w^2}\beta
$$

α Linewidth enhancement factor

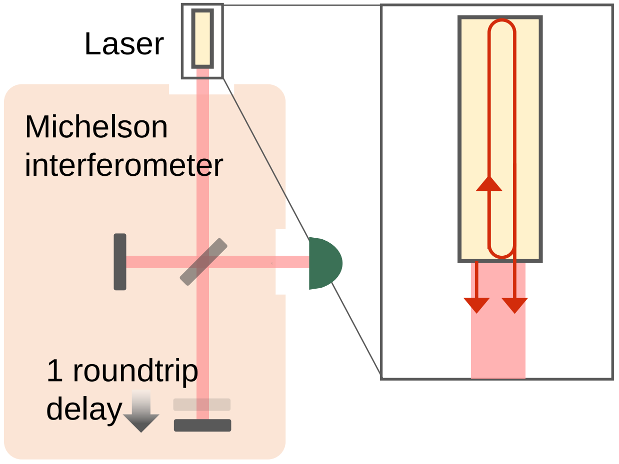
k'' Group velocity dispersion

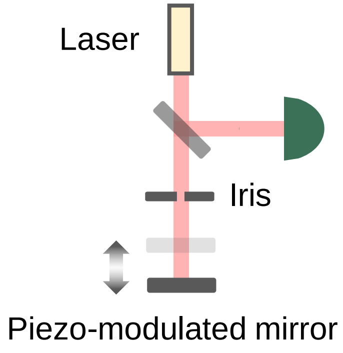
Conditions for the phase instability

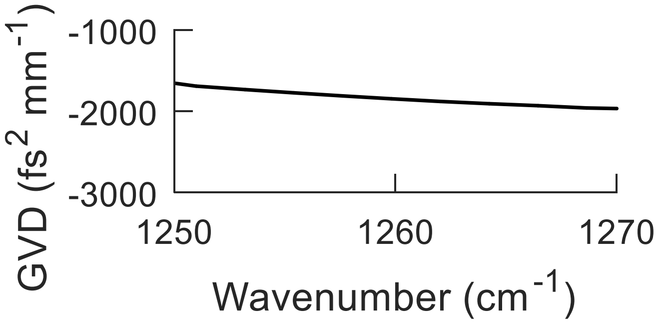


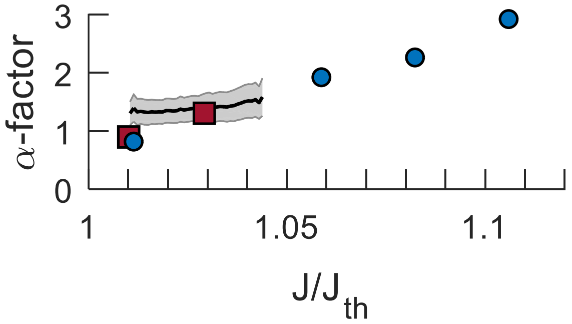
Dispersive
$c_D = -\alpha/2 + \mathrm{GVD} \times \mathrm{const.}$

Nonlinear
$c_{NL} = \alpha/2 + \beta \times \mathrm{const.}$



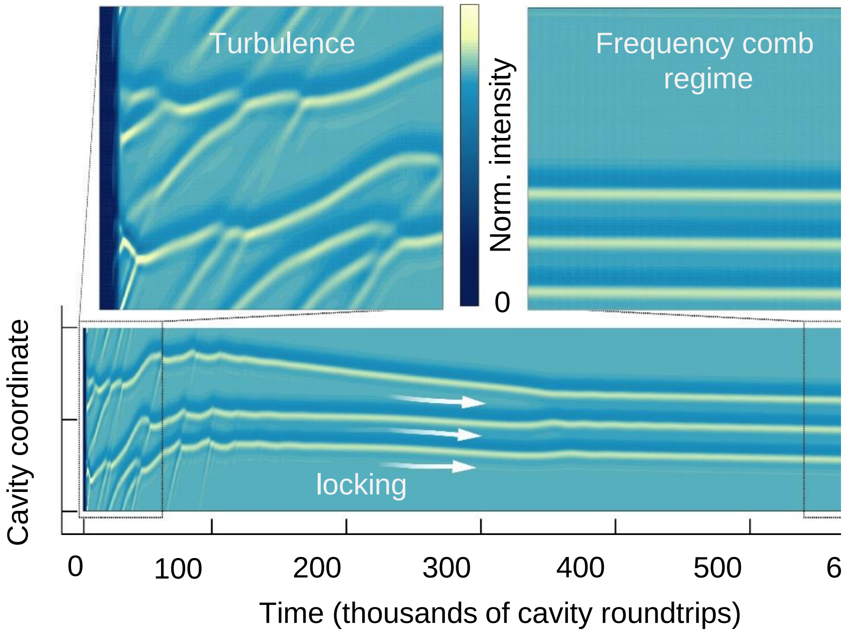


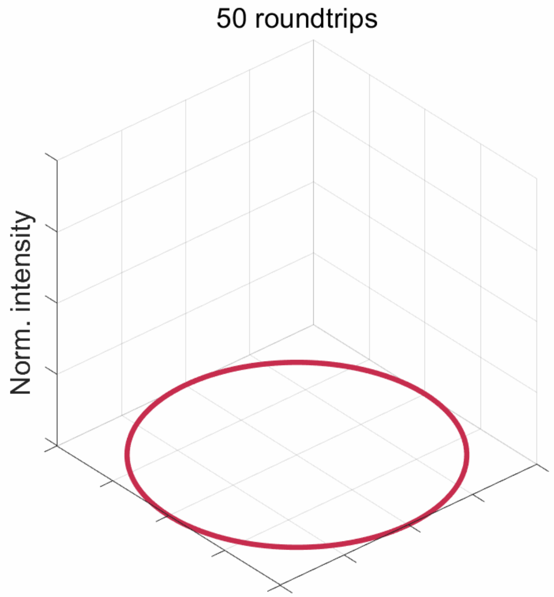




Mechanism of the phase instability

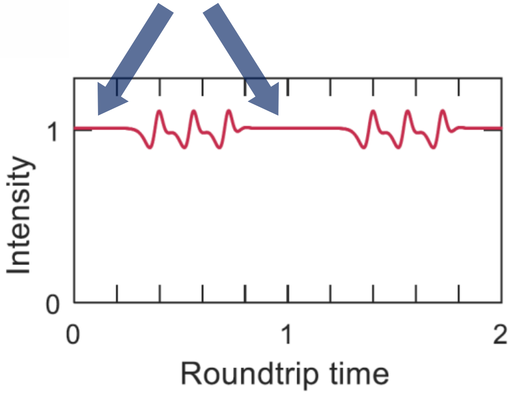
In terms of pattern formation





Classified as: Homoclons

- Shallow localized structures
- Multistable, dependent on the starting conditions
- Connecting to the same plane wave state on both sides

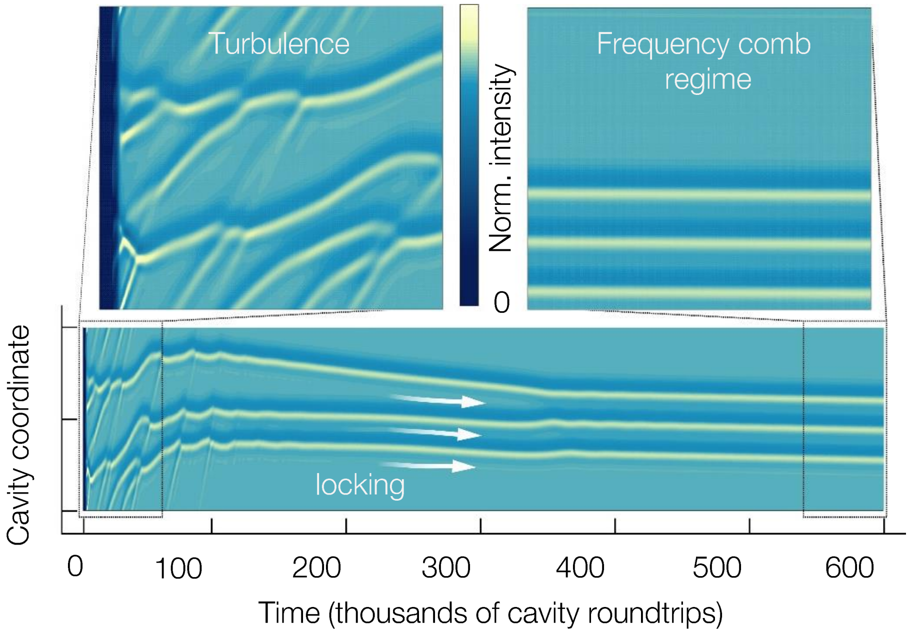


Turbulence is useful!

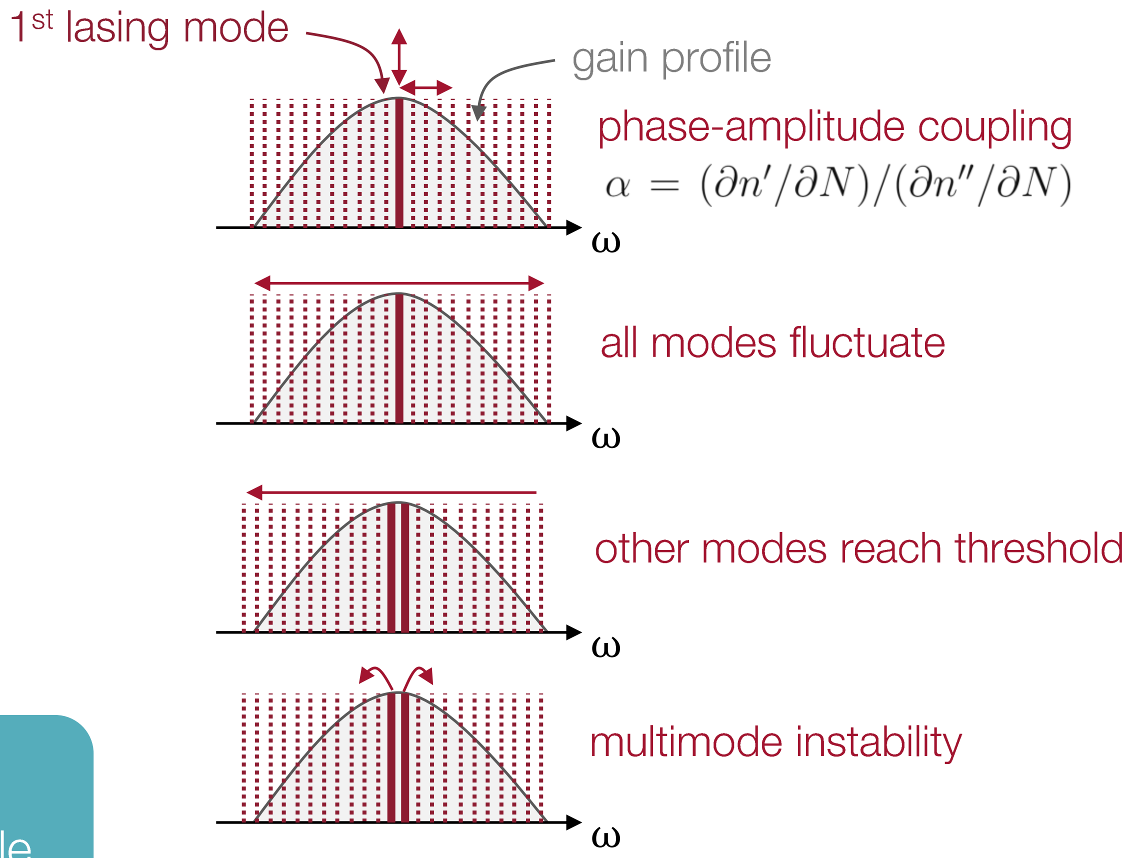
It's what triggers our lasers to operate beyond the single mode regime and eventually generate a frequency comb

Mechanism of the phase instability

In terms of pattern formation



In terms of laser physics

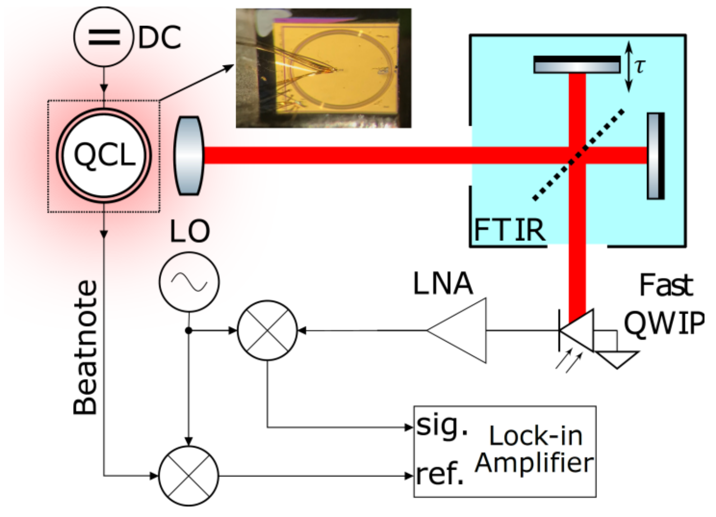


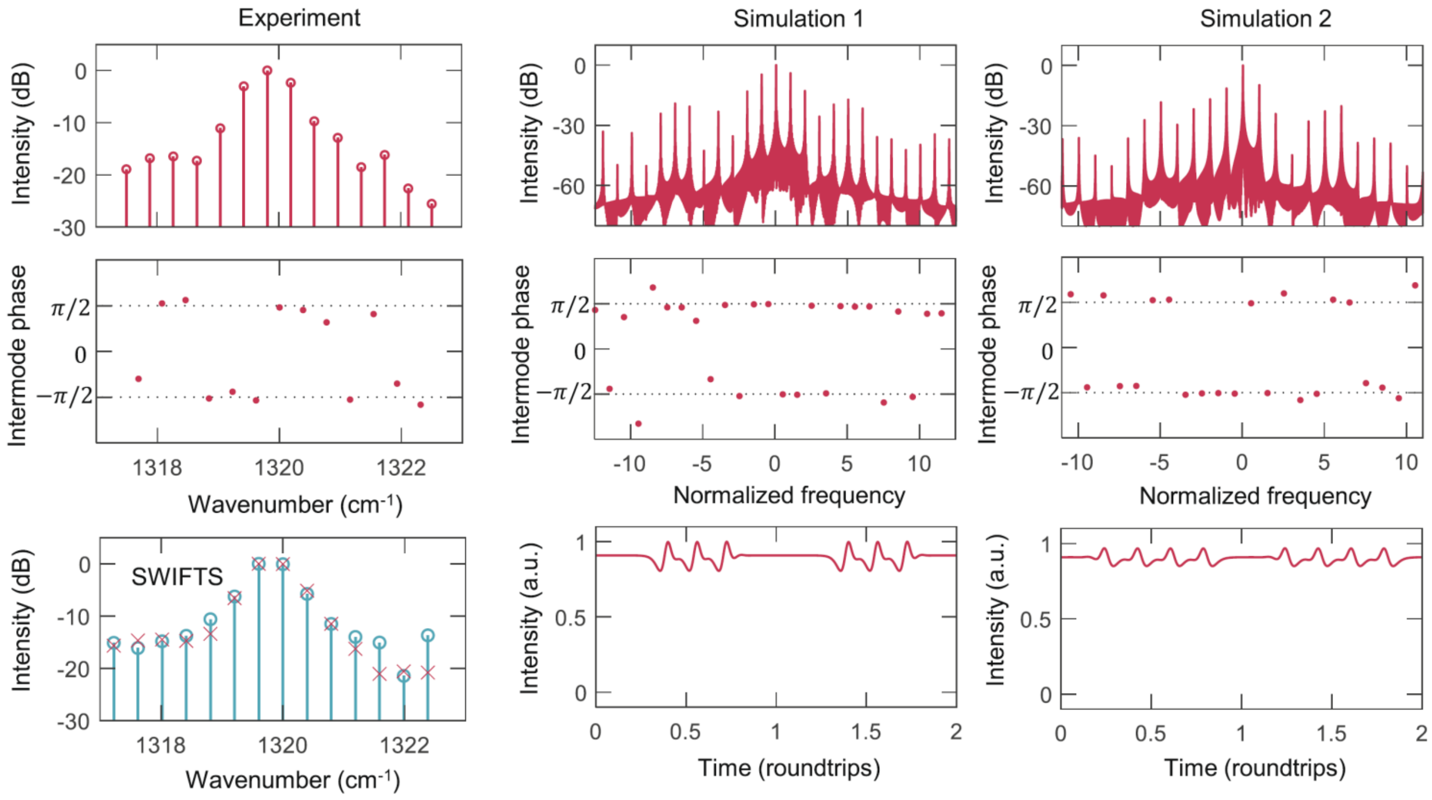
Turbulence is useful!

It's what triggers our lasers to operate beyond the single mode regime and eventually generate a frequency comb

Spectral phase characterization of the homoclons

Shifted Wave Interference Fourier Transform Spectroscopy (SWIFTS)





Multistability:

The *number* of localized structures changes for different seeds of spontaneous emission noise

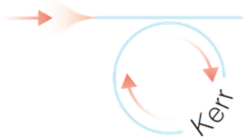
→ Stochastic dependence on the initial conditions

Beyond the CGLE: (our) Generalized Lugiato-Lefever equation

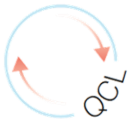
Generalized Lugiato-Lefever Equation

$$\tau_p \partial_t E = \underbrace{E_I}_{\text{Driving field}} + \underbrace{(-1 - i\theta_0)E}_{\text{Damping and detuning}} + \underbrace{(d_r + i d_I)\partial_z^2 E}_{\text{Diffusion and dispersion}} + \underbrace{\mu(1 - i\Delta)(1 - |E|^2)E}_{\text{Gain or loss; optical nonlinearity}}$$

$\mu < 0$ (pumped passive resonator) → LLE



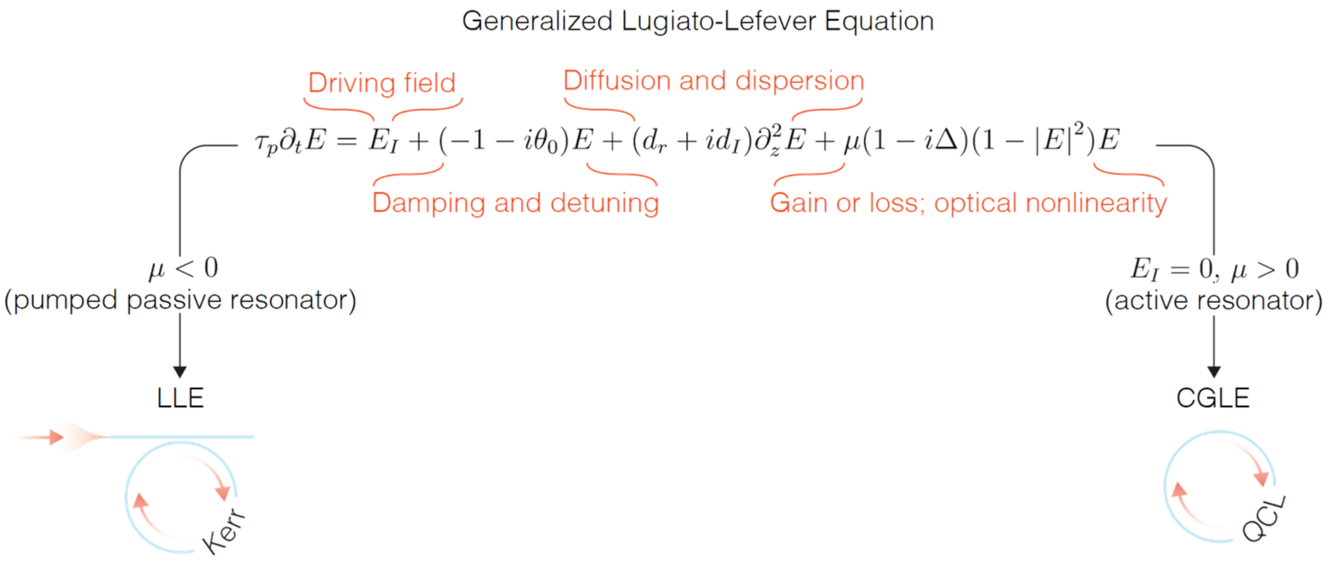
$E_I = 0$, $\mu > 0$ (active resonator) → CGLE



Luigi Lugiato

- Can we complete the Kerr-QCL comb analogy?
- Can we write an LLE for QCLs?
- Can we generate temporal solitons in QCLs?

Beyond the CGLE: (our) Generalized Lugiato-Lefever equation



Damping and Detuning between the injected field, when present, and a cavity resonance

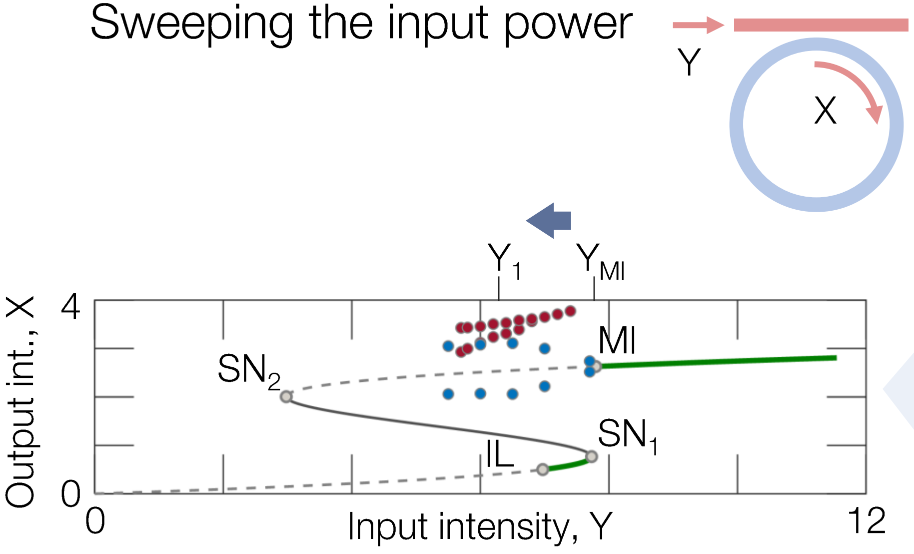
Diffusion and dispersion Account for the frequency dependence of the medium optical response (in d_i add. host medium GVD)

Linear and nonlinear radiation matter interaction Account for the carriers density or population inversion dependence of the optical response close to threshold: μ is the gain (>0) absorption (<0) parameter Δ is the LEF (in Δ add. host medium Kerr nonlinearity) or the atomic detuning

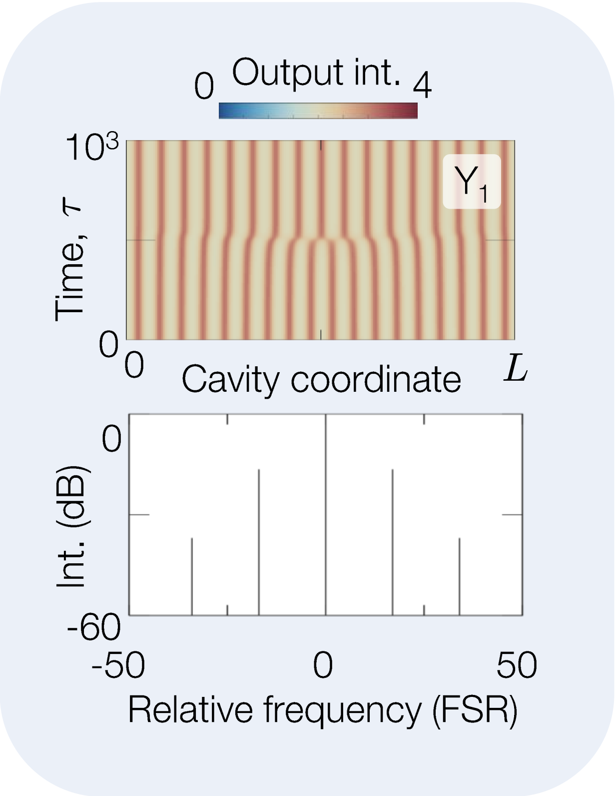
- t and z are the temporal and spatial coordinate along the cavity axis, in a reference frame moving at the light velocity in the cavity; τ_p is the photon life time that defines the typical temporal scale of the system
- Field is scaled as in [L. Lugiato, F. Prati, and M. Brambilla, *Nonlinear Optical Systems* (Cambridge University Press, 2015)]

Pattern control by optical injection

Sweeping the input power



Turing rolls

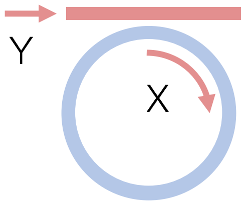


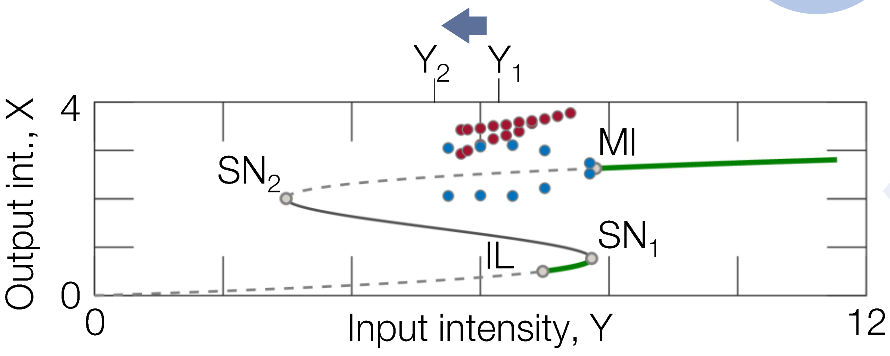
- Global patterns

L. Columbo *et al.*, Phys. Rev. Lett. **126**, 173903 (2021)

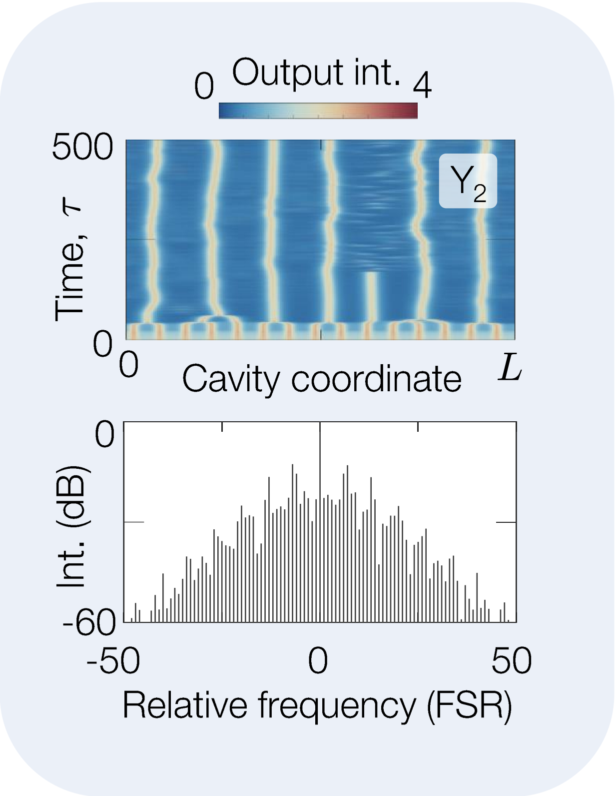
Pattern control by optical injection

Sweeping the input power



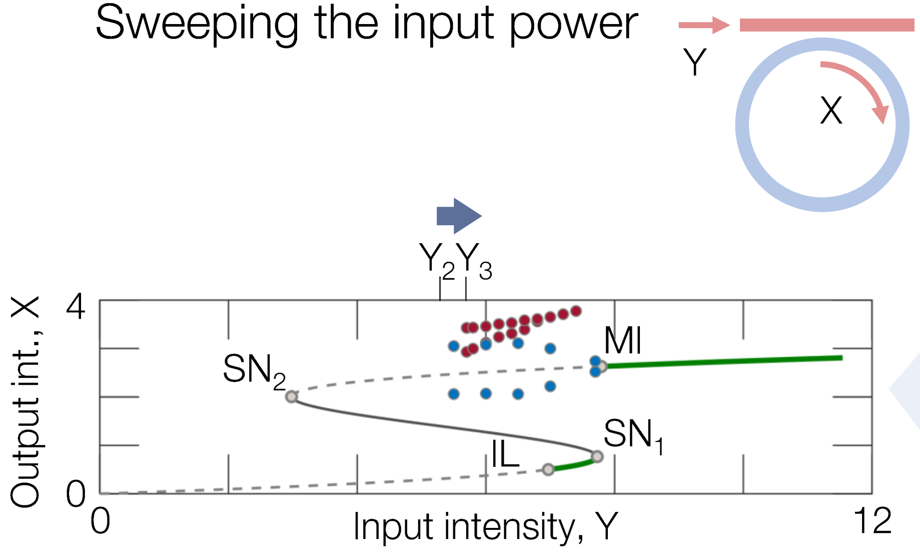


Nonstationary solitons on a turbulent background

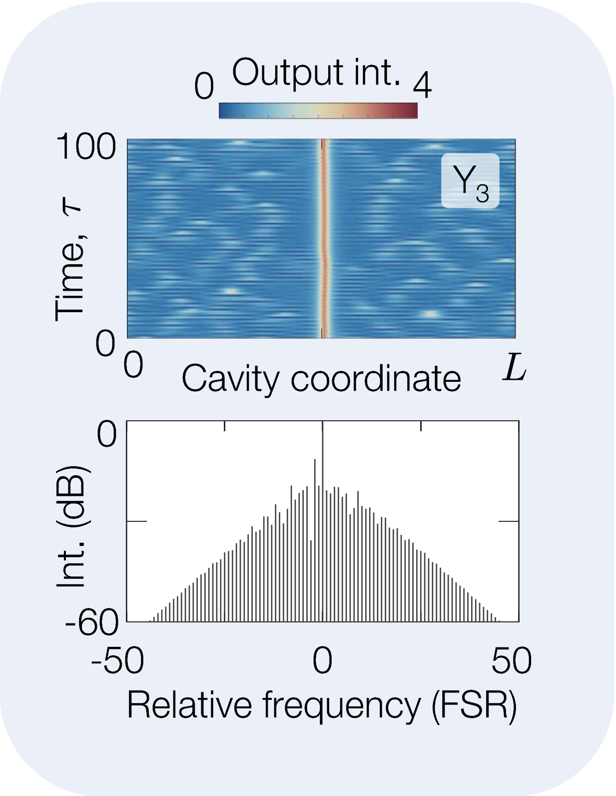


Pattern control by optical injection

Sweeping the input power

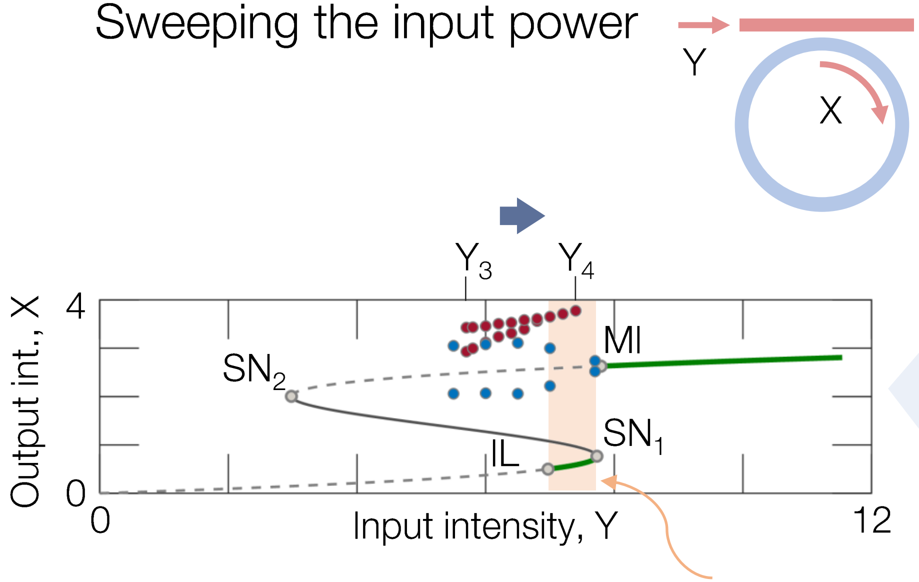


Single soliton on a turbulent background



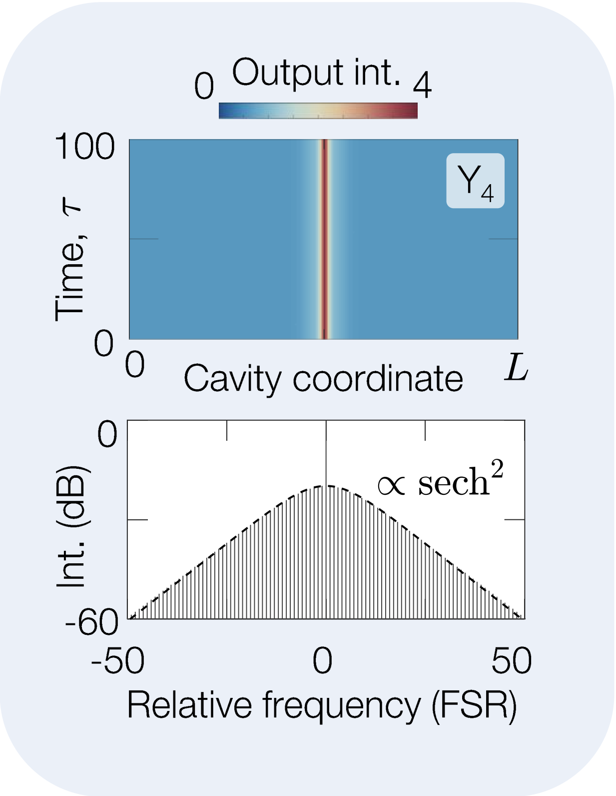
Pattern control by optical injection

Sweeping the input power



Modulational instability on the upper branch *coexisting* with a portion of stable lower branch → favorable for CS

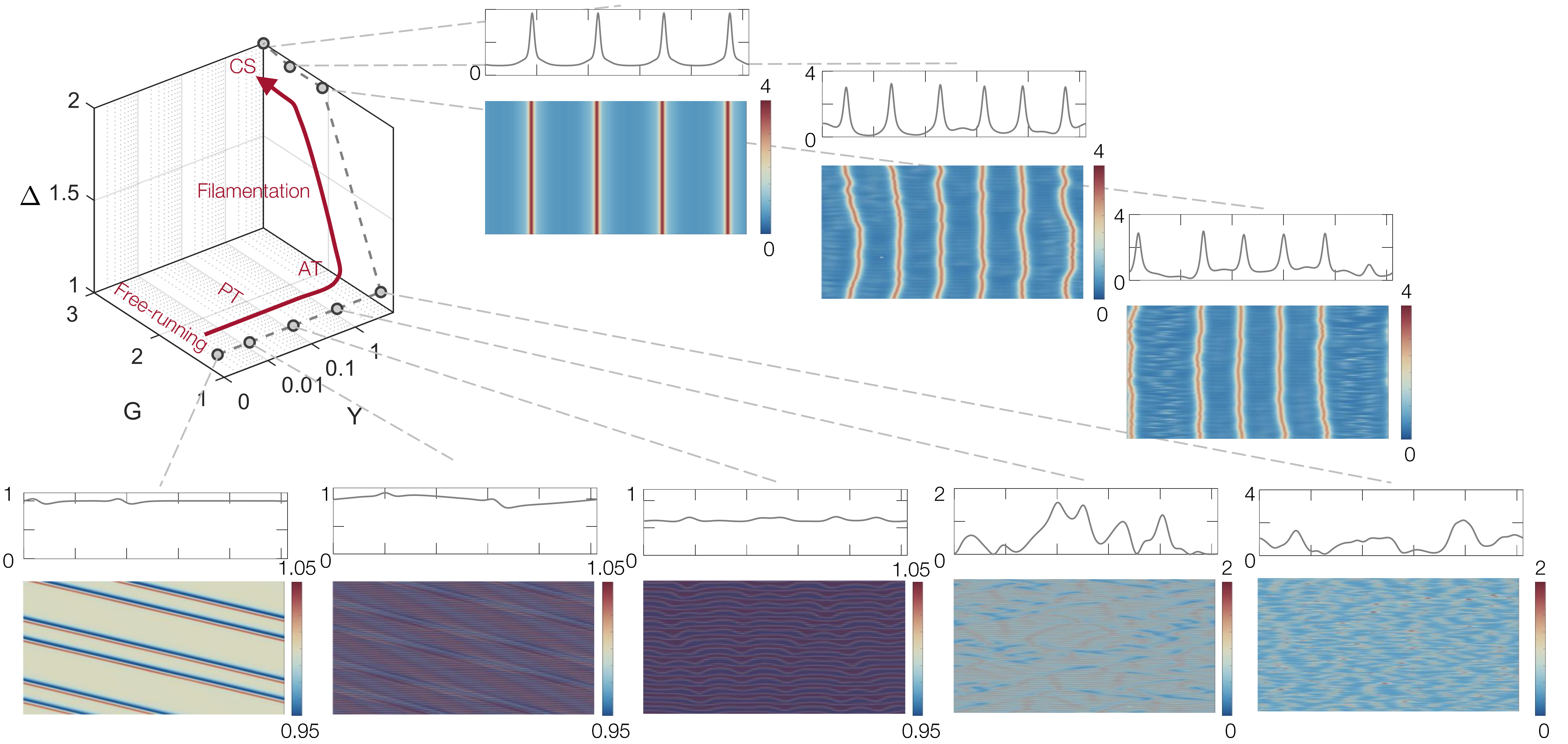
Single stable soliton



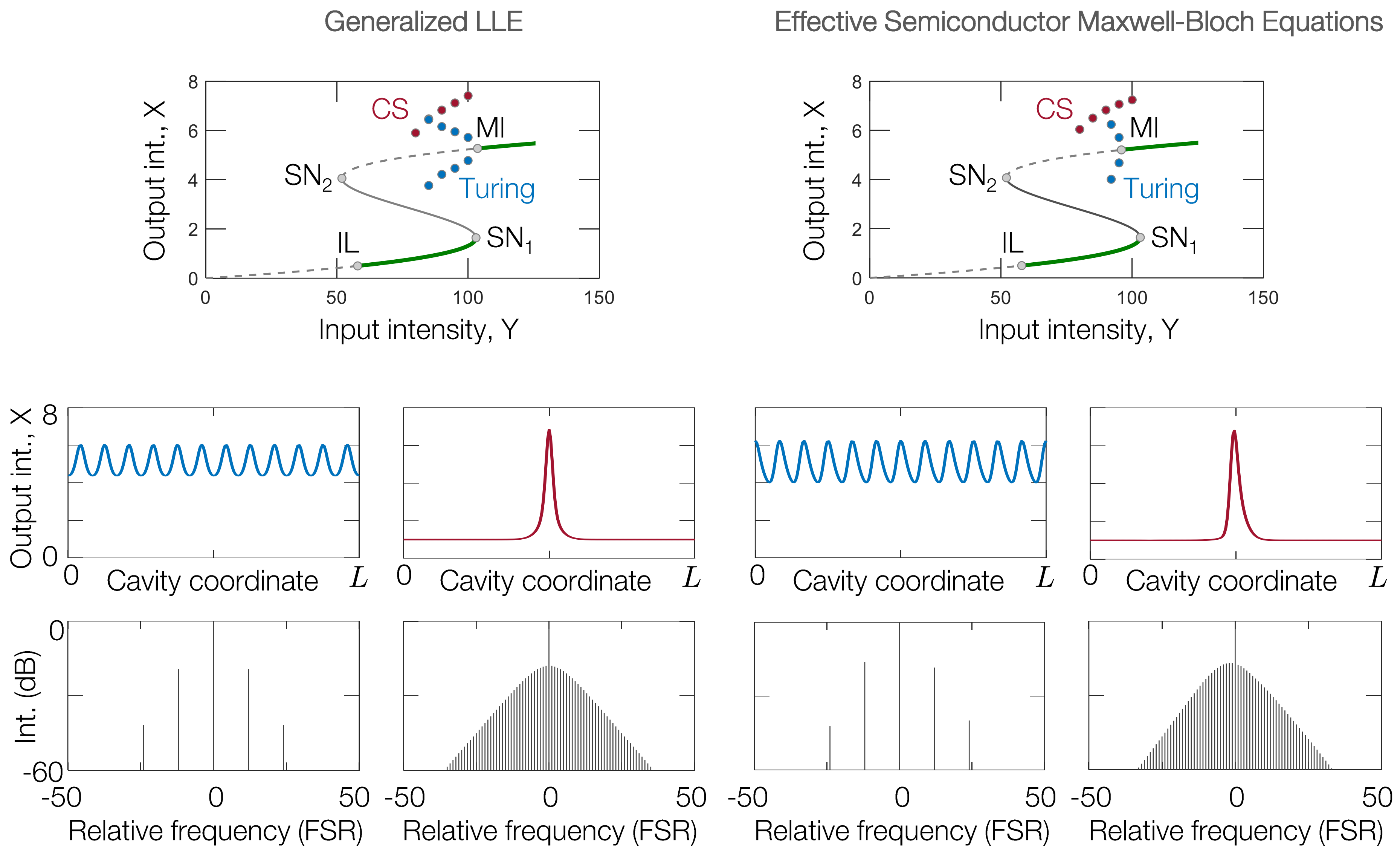
- Localized structures

L. Columbo *et al.*, Phys. Rev. Lett. **126**, 173903 (2021)

Sweeping a trajectory in the parameter space

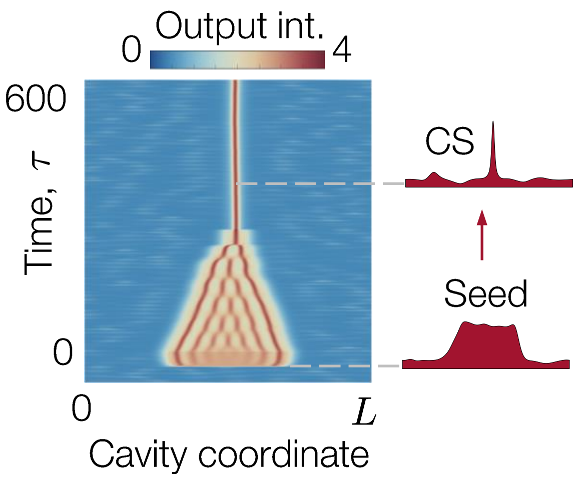


Solitons are predicted also by the full laser model

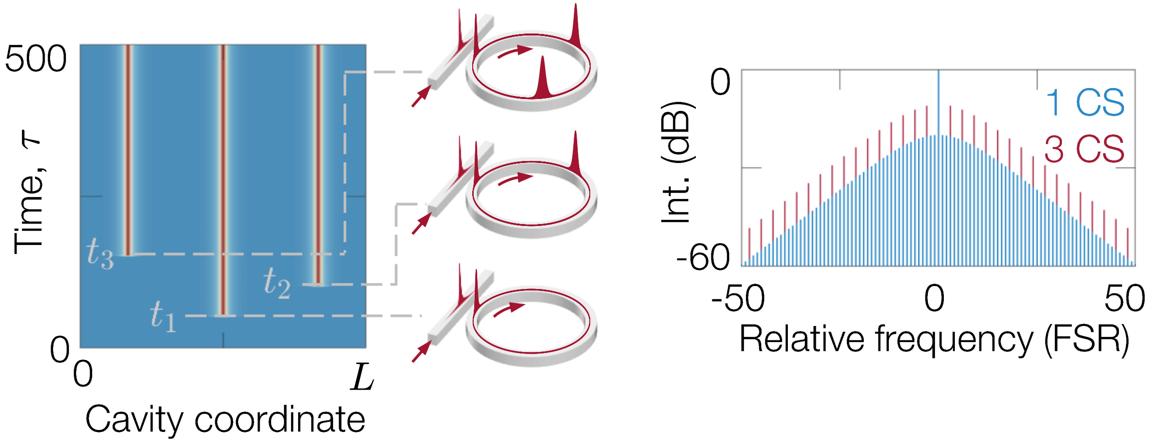


L. Columbo *et al.*, Phys. Rev. Lett. **126**, 173903 (2021)

Independent addressing and manipulation of solitons



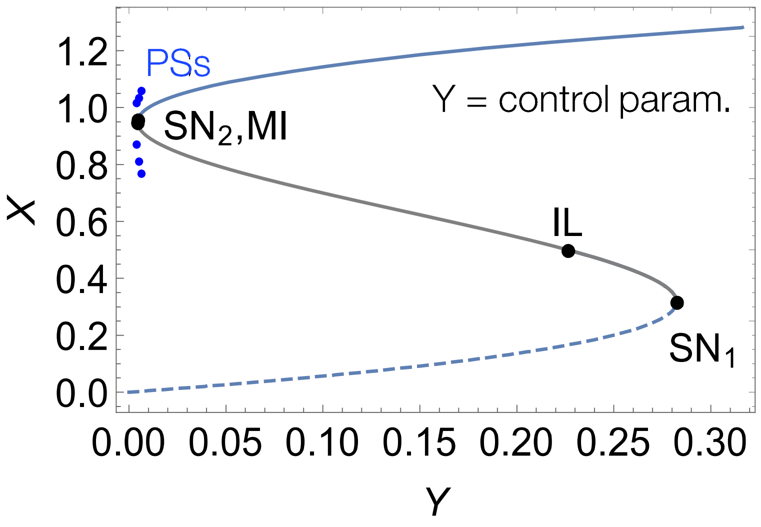
"Writing" a soliton using a long seed pulse

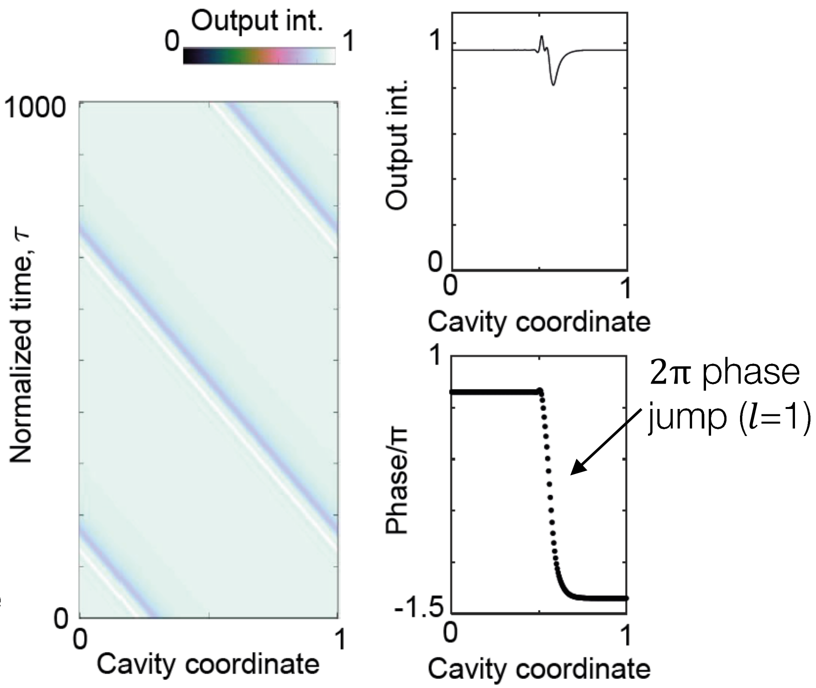


- Multiple solitons can be independently and externally addressed using suitable *sech*-like pulses superimposed to the holding beam
- Controllable harmonic frequency comb spacing

Phase solitons in injected ring QCLs

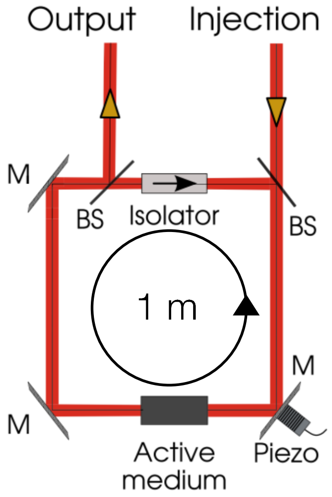
Chiral structures associated to $2l\pi$ $(l \in \mathbb{Z})$ phase kinks



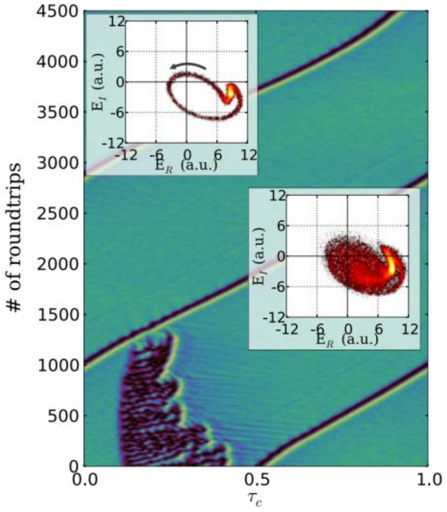


- Lower branch is unstable, upper branch is stable (opposite condition of that for cavity solitons)
- Result from the mismatch between the natural periodicity (free-running laser) and the forcing (external drive)

Observed experimentally in driven bipolar semiconductor ring laser

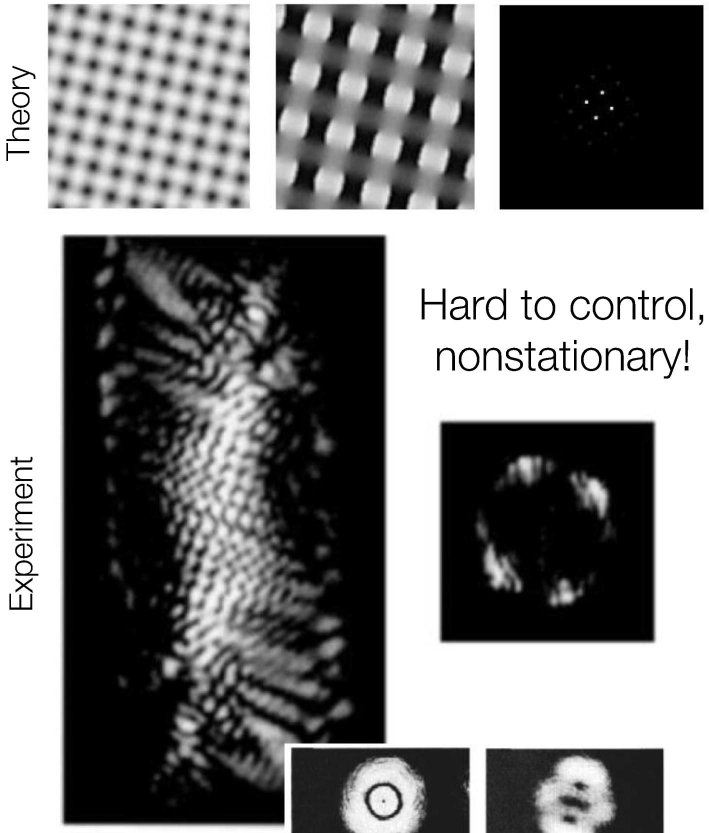


T_1 ~ 1 ns

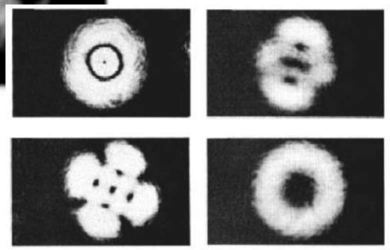


Controlling the formation of chiral structures in lasers with metasurfaces

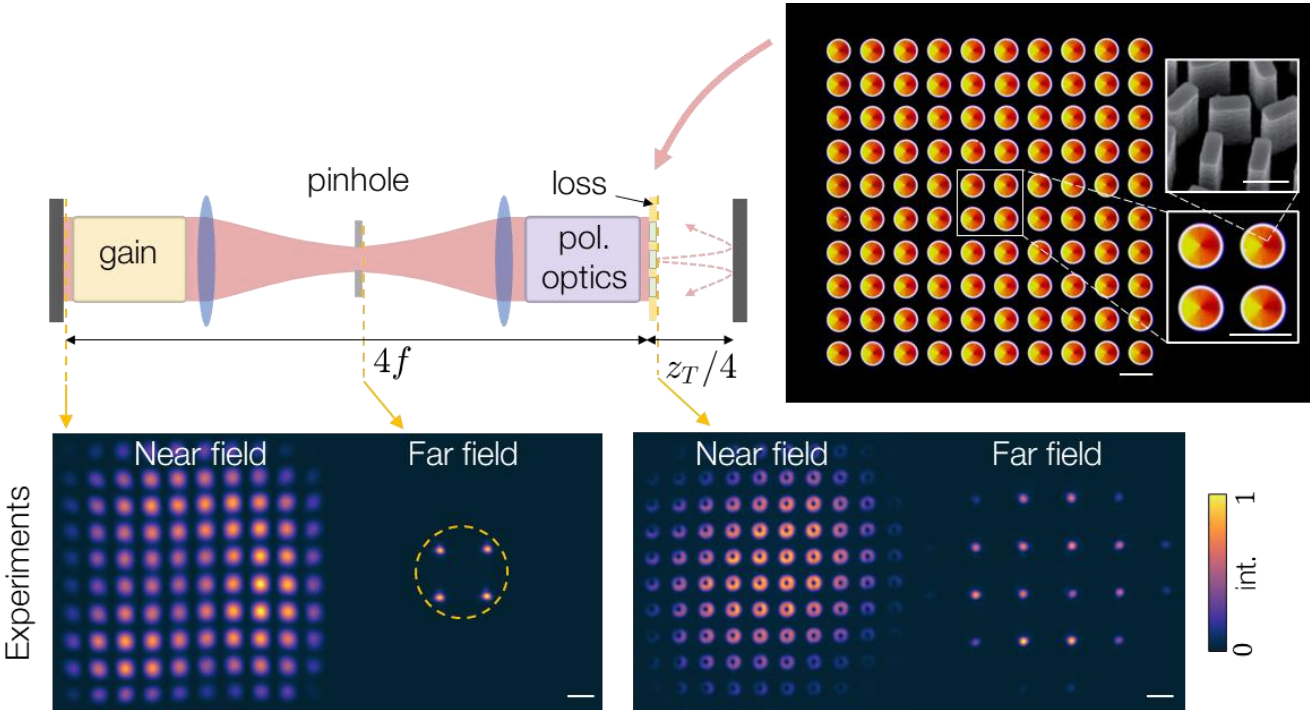
Spontaneous patterns in lasers



Hard to control, nonstationary!



Vortex laser arrays with 100 metasurfaces



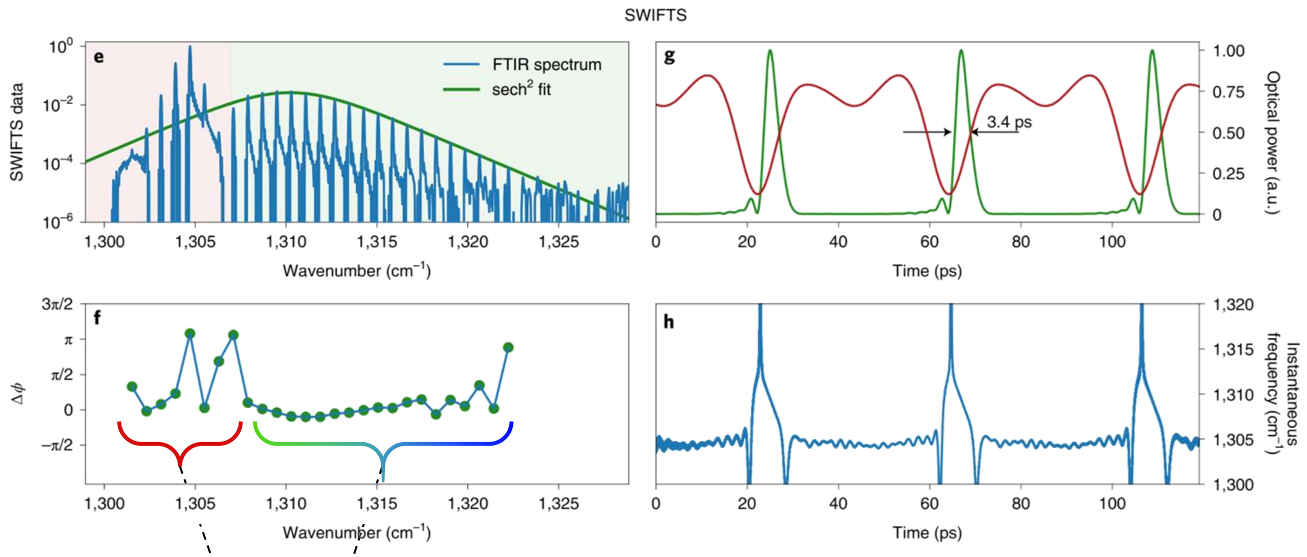
- Tunable topological solutions
- Strong coupling
- Defect healing

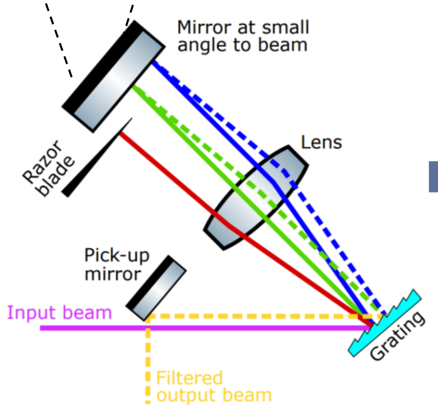
K. Staliunas *et al., Transverse Patterns in Nonlinear Optical Resonators*, Springer (2003)

M. Piccardo *et al.*, Nat. Photon. 16, 359 (2022)

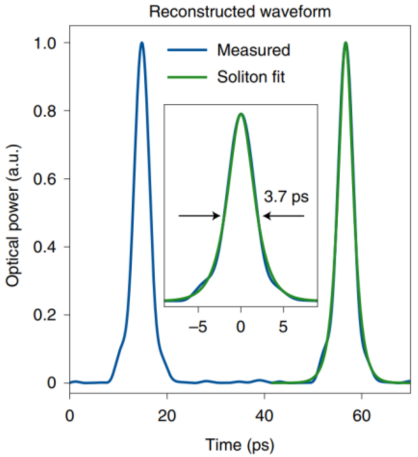
Solitons in free-running ring QCLs

Spectral selection of in-phase modes

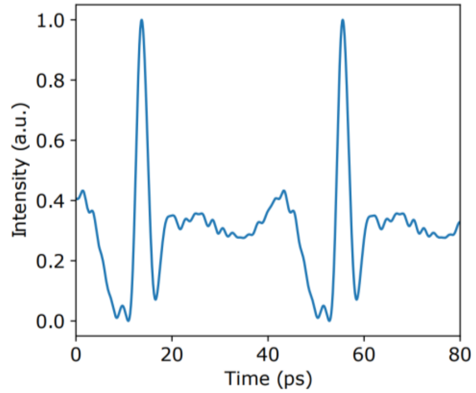




Filtered pulse

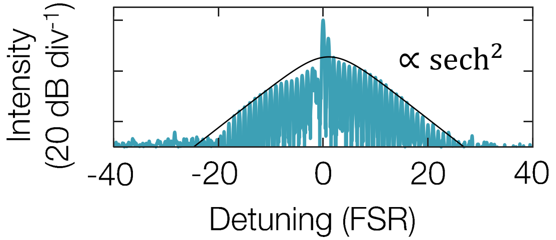


Total waveform

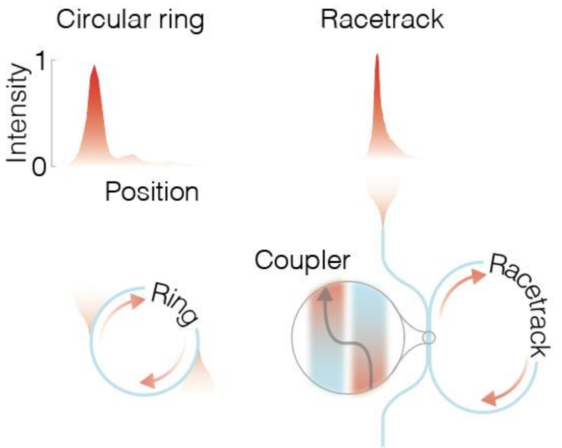


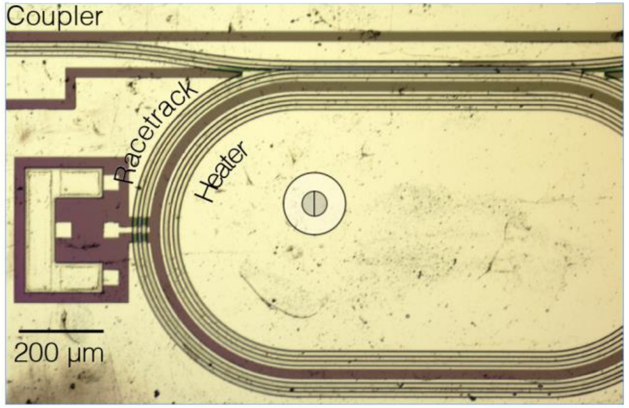
- No external pump → no phase symmetry breaking
- Generation mechanism of bright dissipative Kerr solitons (*sech*-like) in these conditions is not completely clear

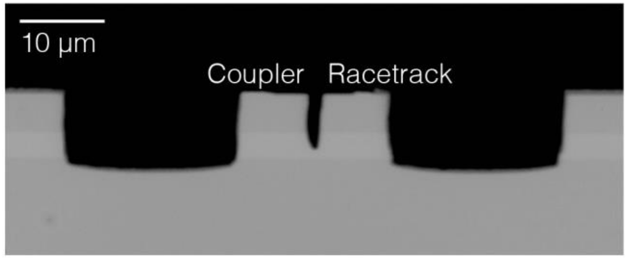
Similar spectra are being studied at Harvard & TU Vienna:



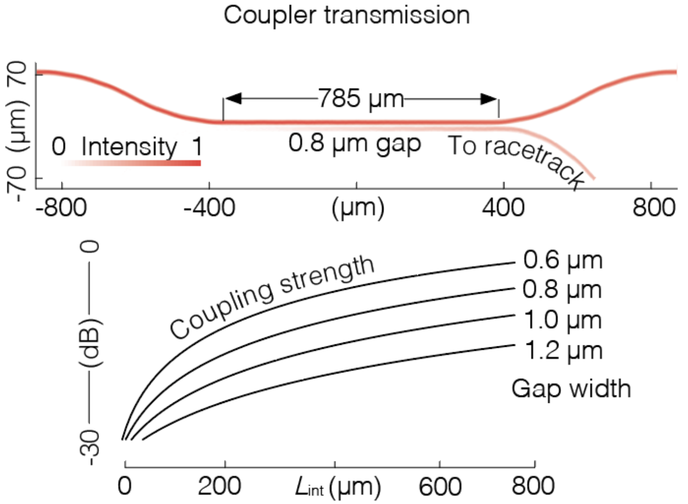
Towards demonstrating injected ring QCLs



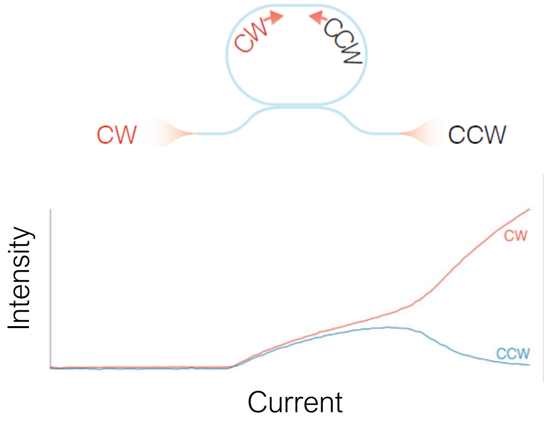


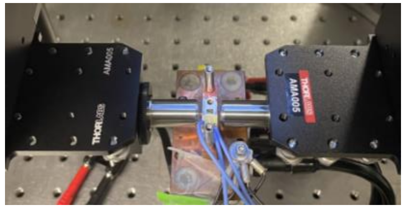


Evanescent wave directional coupler

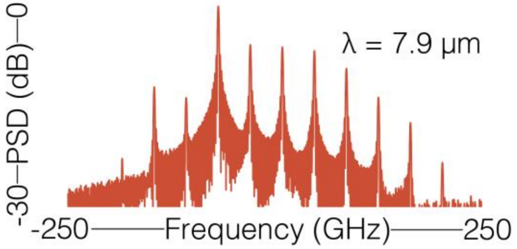


Spontaneous symmetry breaking observed



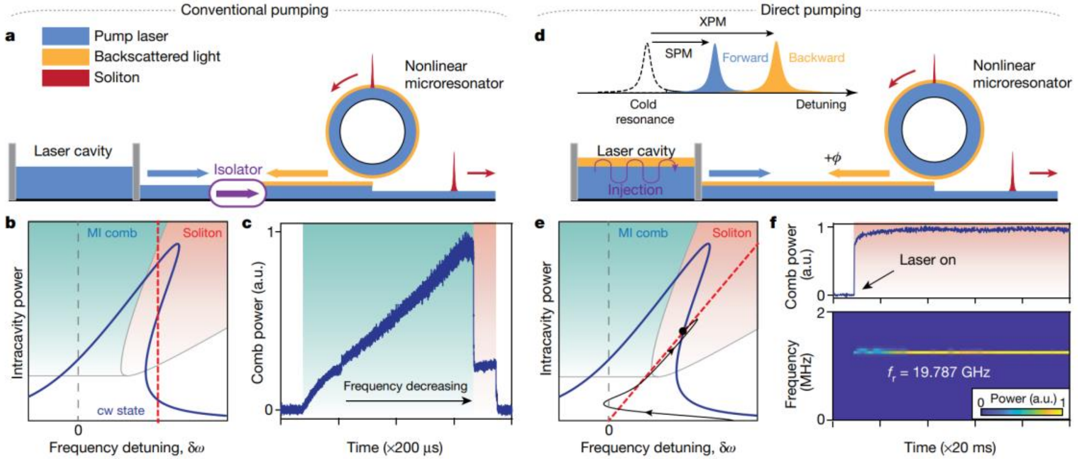


Ring frequency comb with 10's of mW of outcoupled power



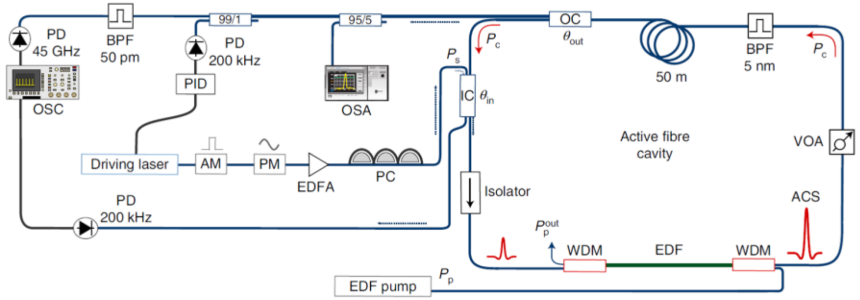
Perspectives

Turnkey soliton generation in a pump-microresonator system



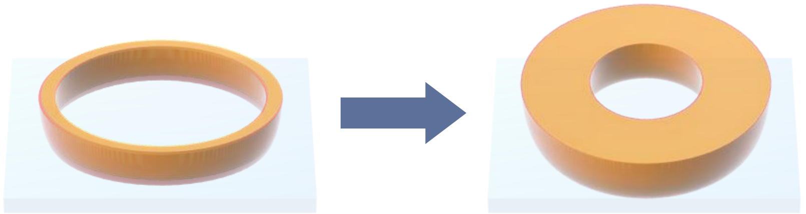
- Enable feedback from QCL ring to pump → self-tuning?

Hybrid fiber system shows that soliton dynamics depends on driving CW/pulse



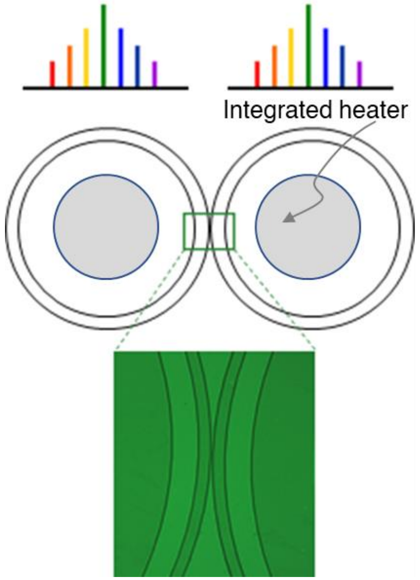
- Injected QCL ring driven by long pulses → spontaneous CS?

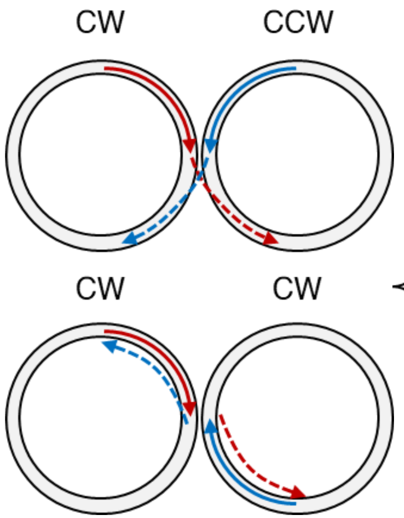
Spatiotemporal patterns in broad area ring QCLs



- Complicated to make a VECSEL in ring cavity
  → broad area monolithic ring with fast gain
- GVD favorable for solitons? Transverse mode interplay?

Coupled ring QCLs





Coupled forced CGLEs

$$\begin{cases} \dfrac{dE_1}{d\tau} = c_1 E_2 + \mathrm{CGLE}(E_1) \\ \dfrac{dE_2}{d\tau} = c_2 E_1 + \mathrm{CGLE}(E_2) \end{cases}$$

B. Shen *et al.*, Nature 582, 365 (2020); N. Englebert *et al.*, Nat. Photon. 15, 536 (2021)