Condensation of optical waves and the role of disorder

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<u>*Theory*</u>

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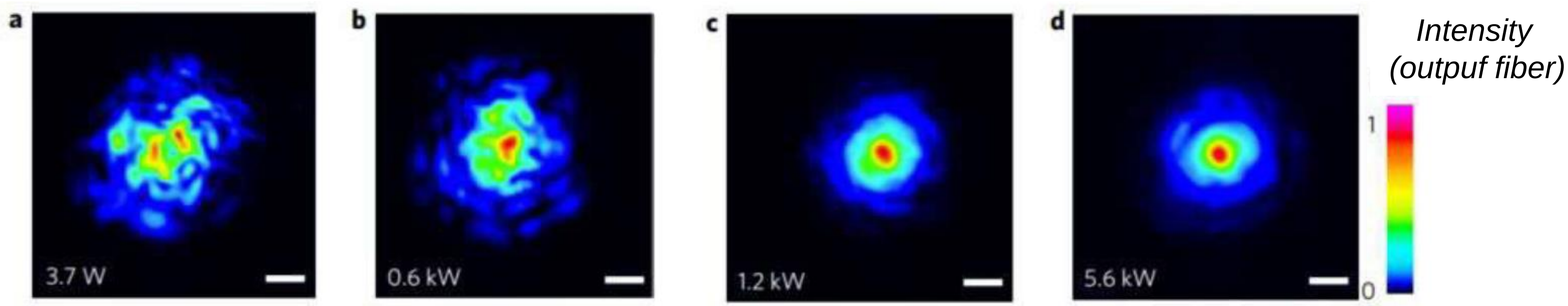
Waves & Complexity school -- Porquerolles, May 15-20, 2022

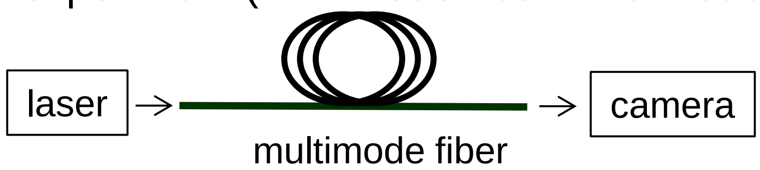
Experiment that motivated our study:
Spatial beam self-cleaning in a multimode optical fiber



Limoges – Dijon – Brescia consortium
Krupa, Tonello, Shalaby, Fabert, Barthélémy, Millot, Wabnitz, Couderc, Nature Photonics 11, 237 (2017)

Cornell Univ.
Wright, Liu, Nolan, Li, Christodoulides, Wise, Nature Photonics 10, 771 (2016)

- Conceptually simple experiment (multimode fiber: 120 modes, ~10-20 meters, ns pulses)

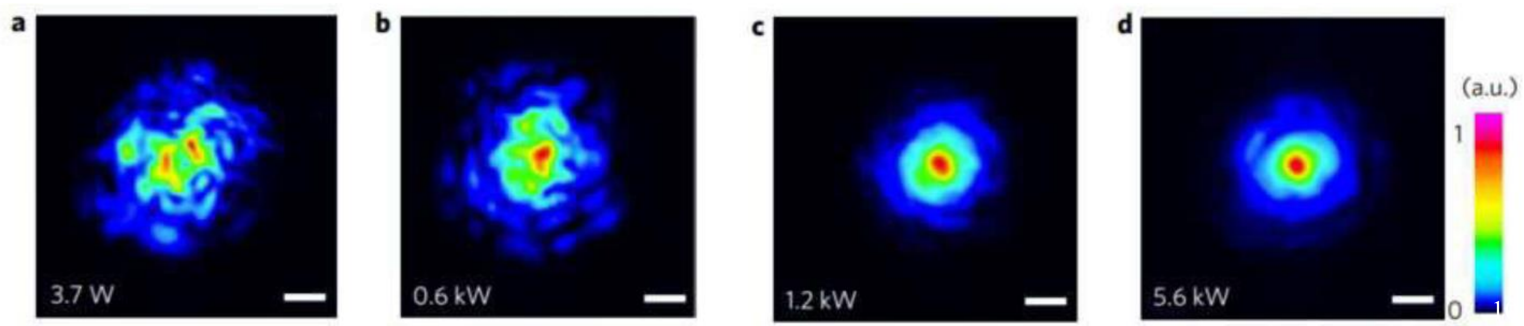


- Beam cleaning is due to a conservative Kerr nonlinearity (i.e. four-wave interaction)
  → no gain/loss, no thermal heat bath
  → can be described by a generalized NLS equation

→ Is this effect of beam self-cleaning related to wave condensation ?

→ Impact of disorder : light propagation is affected by bending and stretching of the fiber introduce linear random coupling among the modes

Experimental results that motivated our study: Spatial beam self-cleaning in a multimode optical fiber



Outline of the talk

1.- Reminder of wave condensation in a homogeneous system

2.- Reminder of wave condensation in a confined trapping potential

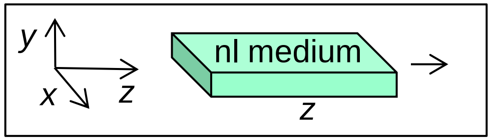
3.- Impact of a *weak disorder* on wave condensation
→ acceleration of the rate of thermalization & condensation

4.- Experiments: Observation of the transition to light condensation

5.- Impact of *strong disorder* (genuine random mode coupling)
→ unveils the interplay of nonlinearity and disorder

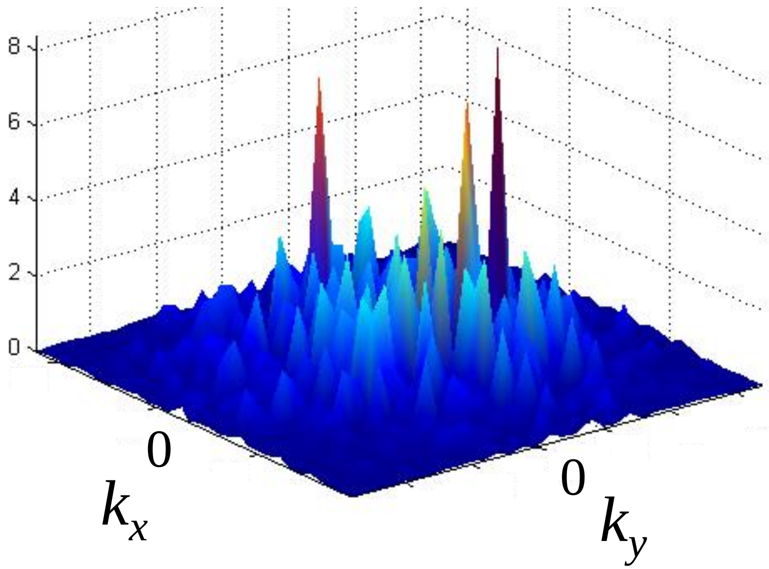
Reminder: Condensation of classical waves

2D NLS $$i\partial_z \psi = -\nabla_\perp^2 \psi + |\psi|^2 \psi$$

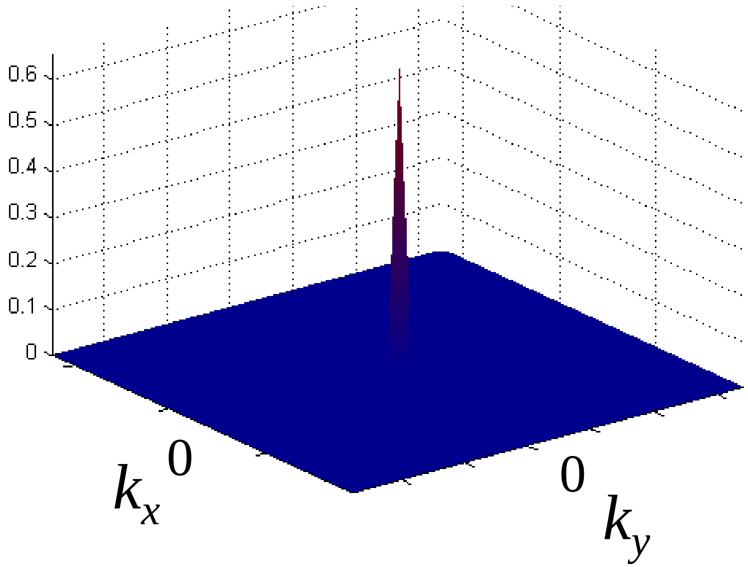


$|\tilde{\psi}(z=0,k)|^2$

10^{-4} Initial condition (random spectrum)



Thermodynamic Equilibrium

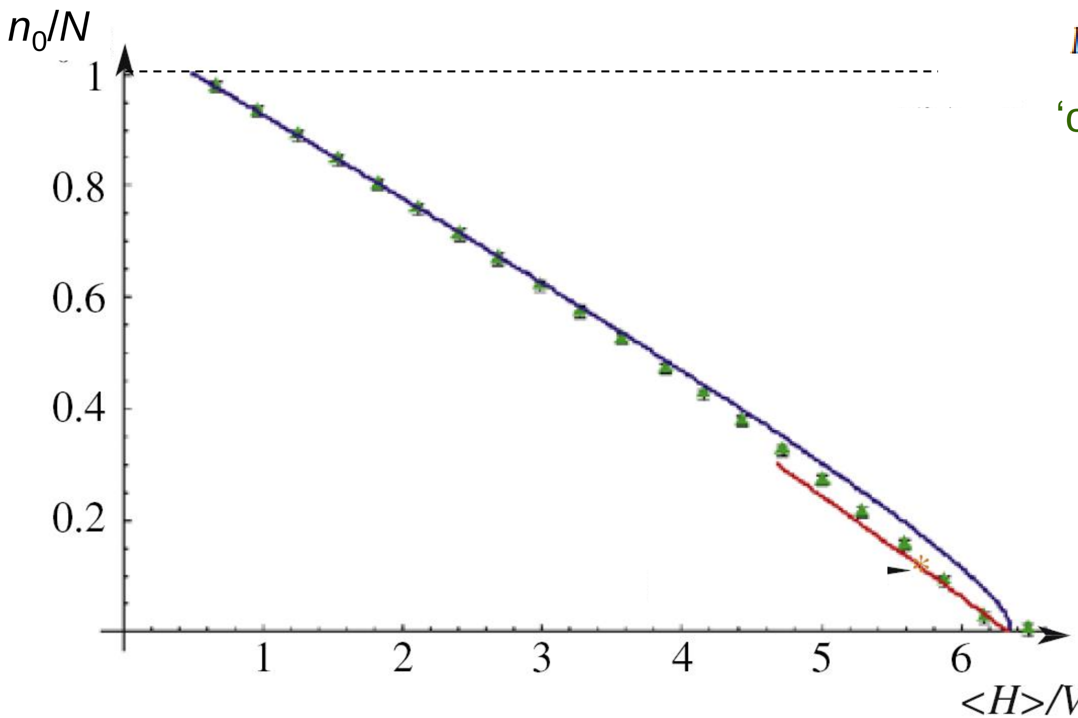


→ **Condensation results from the natural thermalization to the thermodynamic equilibrium state**

Zakharov, Nazarenko -- Physica D (2005)
Connaughton, Josserand, Picozzi, Pomeau, Rica -- PRL (2005)
Krstulovic, Brachet -- PRL (2011) ; PRE (2011)

Reminder: Wave condensation is a phase transition

‘optics language’: Fraction of power in the fundamental mode



$$N = \int |\psi|^2 \mathrm{d}^D \boldsymbol{x} = const$$

‘optics language’: power

Energy or Hamiltonian (‘temperature’)

Connaughton, Josserand, Picozzi, Pomeau, Rica -- PRL (2005)

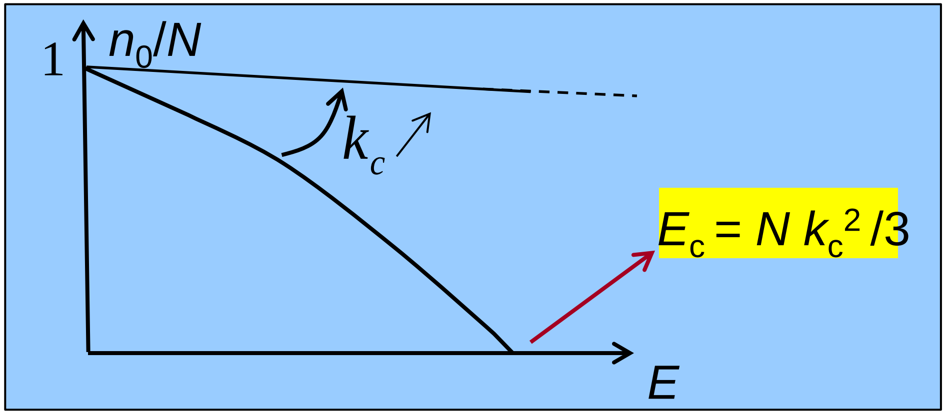
Reminder: Wave condensation requires a frequency cut-off

$$\begin{cases} i\partial_z\psi = -\nabla^2\psi + |\psi|^2\psi \\ N = \int n_k dk \\ E = \int k^2 n_k dk \end{cases} \xrightarrow[\text{Rayleigh-Jeans equilibrium spectrum}]{\text{Wave turbulence th.}} \begin{cases} n_k^{eq} = \dfrac{T}{k^2-\mu} \\ \lambda_c \propto 1/\sqrt{-\mu} \end{cases}$$

k_c: UV catastrophe

$$3\text{D} \begin{cases} N = 4\pi T k_c \left[1 - \dfrac{\sqrt{-\mu}}{k_c}\,\mathrm{arctg}\left(\dfrac{k_c}{\sqrt{-\mu}}\right)\right] \\ \mu \to 0 \ \text{ for } \ T = T_c \neq 0 \end{cases}$$

$$2\text{D} \begin{cases} N = \pi T \log(1 - k_c^2/\mu) \\ \mu \to 0 \ \text{ for } \ T = 0 \end{cases}$$



→ Only transient process of ***far from equilibrium condensation*** is accessible in an optical exp.

Sun, Jia, Barsi, Rica, Picozzi, Fleischer, Nat. Phys. 8, 471 (2012)

→ *Santic, Fusaro, Salem, Garnier, Picozzi, Kaiser PRL 120, 055301 (2018)*

→ **However: UV catastrophe can be regularized by introducing a frequency cut-off** k_c

→ **effective physical frequency cut-off** k_c **arises in an optical waveguide**

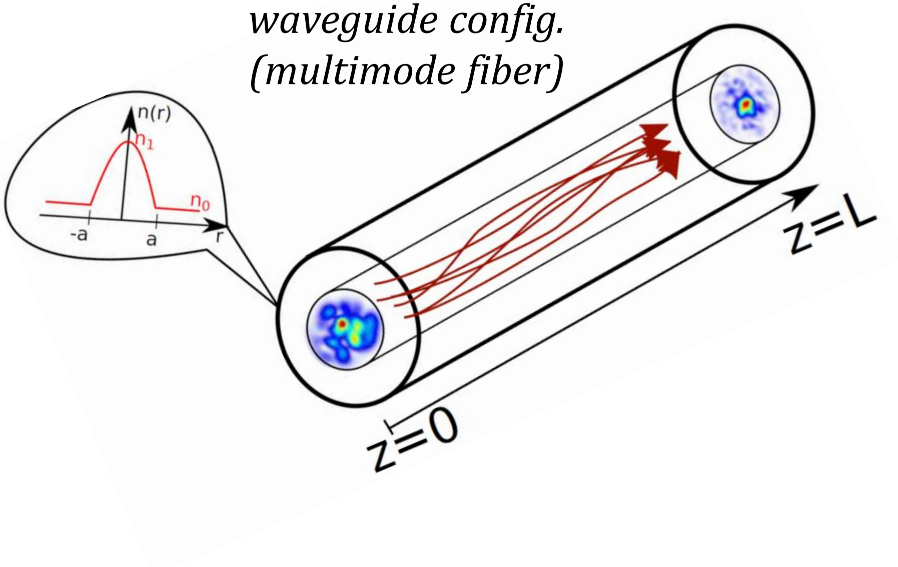
Reminder: Wave condensation in a trap *V(r)*

$$i\partial_z\psi = -\alpha\nabla^2\psi + V(\mathbf{r})\psi + \gamma|\psi|^2\psi$$

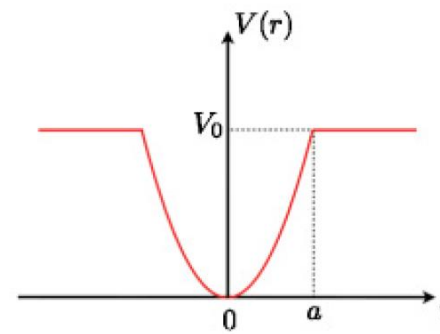
(D = 2)

Speckle beam guided by a truncated parabolic potential

waveguide config.
(multimode fiber)



$$V(\mathbf{r}) = k_0\left(\frac{n_1^2 - n^2(\mathbf{r})}{2n_0}\right)$$



$$N = \int |\psi|^2 d\mathbf{r}$$

$$H = \underbrace{\int \alpha|\nabla\psi|^2 d\mathbf{r} + \int V(\mathbf{r})|\psi|^2 d\mathbf{r}}_{\text{Linear, } E} + \underbrace{\frac{\gamma}{2}\int |\psi|^4 d\mathbf{r}}_{\text{Nonlinear, } U}$$

Finite number of modes of the waveguide introduces an ‘effective frequency cut-off’

$\longrightarrow$ Revisit wave turbulence theory with a trapping potential

Reminder: Wave condensation in a trap *V(r)*

$$i\partial_z\psi = -\alpha\nabla^2\psi + V(\mathbf{r})\psi + \gamma|\psi|^2\psi \qquad \psi(\mathbf{r},z) = \sum_m c_m(z)u_m(\mathbf{r})\exp(-i\beta_m z)$$

$$i\partial_z a_m = \beta_m a_m + \gamma\sum_{p,q,s} W_{mpqs}a_p a_q^* a_s \qquad a_m(z) = c_m(z)\exp(-i\beta_m z)$$

$$W_{mpqs} = \int u_m^*(\mathbf{r})u_p(\mathbf{r})u_q^*(\mathbf{r})u_s(\mathbf{r})d\mathbf{r} \qquad n_m(z) = \left\langle\left|\int \psi(\boldsymbol{r},z)\,u_m^*(\boldsymbol{r})\,d\boldsymbol{r}\right|^2\right\rangle = \langle|c_m(z)|^2\rangle$$

WT theory: S. Nazarenko, *Wave Turbulence* (Springer, 2011)

Weakly nonlinear regime & Continuous limit: $V_0/\beta_0 \gg 1$

$$\partial_z\tilde{n}_\kappa = \frac{4\pi\gamma^2}{\beta_0^6}\iiint d\kappa_1 d\kappa_2 d\kappa_3\,\delta\big(\tilde{\beta}_{\kappa_1}+\tilde{\beta}_{\kappa_3}-\tilde{\beta}_{\kappa_2}-\tilde{\beta}_\kappa\big)$$
$$\times\left|\widetilde{W}_{\kappa\kappa_1\kappa_2\kappa_3}\right|^2\tilde{n}_\kappa\tilde{n}_{\kappa_1}\tilde{n}_{\kappa_2}\tilde{n}_{\kappa_3}\big(\tilde{n}_\kappa^{-1}+\tilde{n}_{\kappa_2}^{-1}-\tilde{n}_{\kappa_1}^{-1}-\tilde{n}_{\kappa_3}^{-1}\big)$$
$$+\frac{8\pi\gamma^2}{\beta_0^2}\int d\kappa_1\,\delta\big(\tilde{\beta}_{\kappa_1}-\tilde{\beta}_\kappa\big)\left|\widetilde{U}_{\kappa\kappa_1}(\tilde{n})\right|^2(\tilde{n}_{\kappa_1}-\tilde{n}_\kappa)$$

$$\widetilde{U}_{\kappa\kappa_1}(\tilde{n}) = \frac{1}{\beta_0^2}\int d\kappa'\,\widetilde{W}_{\kappa\kappa_1\kappa'\kappa'}\tilde{n}_{\kappa'}$$

$$\tilde{\beta}_\kappa = \beta_{[k/\beta_0]}$$
$$\tilde{n}_\kappa(z) = n_{[k/\beta_0]}(z)$$
$$\boldsymbol{\kappa} = \beta_0(m_x, m_y)$$

Aschieri, Garnier, Michel, Doya, Picozzi -- PRA (2011)

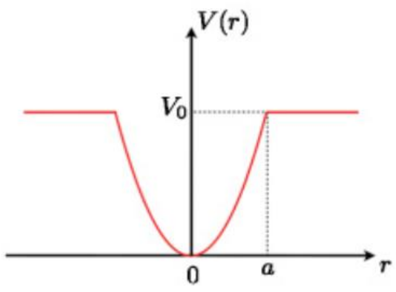
Reminder: Wave condensation in a trap *V(r)*

$$\partial_z \tilde{n}_\kappa = \frac{4\pi\gamma^2}{\beta_0^6} \iiint d\kappa_1 d\kappa_2 d\kappa_3 \delta\big(\tilde{\beta}_{\kappa_1} + \tilde{\beta}_{\kappa_3} - \tilde{\beta}_{\kappa_2} - \tilde{\beta}_\kappa\big)$$
$$\times \big|\widetilde{W}_{\kappa\kappa_1\kappa_2\kappa_3}\big|^2 \tilde{n}_\kappa \tilde{n}_{\kappa_1} \tilde{n}_{\kappa_2} \tilde{n}_{\kappa_3} \big(\tilde{n}_\kappa^{-1} + \tilde{n}_{\kappa_2}^{-1} - \tilde{n}_{\kappa_1}^{-1} - \tilde{n}_{\kappa_3}^{-1}\big)$$
$$+ \frac{8\pi\gamma^2}{\beta_0^2} \int d\kappa_1 \delta\big(\tilde{\beta}_{\kappa_1} - \tilde{\beta}_\kappa\big) \big|\widetilde{U}_{\kappa\kappa_1}(\tilde{n})\big|^2 (\tilde{n}_{\kappa_1} - \tilde{n}_\kappa)$$

$N = \beta_0^{-2} \int d\boldsymbol{\kappa} \tilde{n}_{\boldsymbol{\kappa}}$ = *const* ‘power’

$E = \beta_0^{-2} \int d\boldsymbol{\kappa} \tilde{\beta}_{\boldsymbol{\kappa}} \tilde{n}_{\boldsymbol{\kappa}}$ = *const* ‘energy’

$\mathcal{S}(z) = \beta_0^{-2} \int d\boldsymbol{\kappa} \ln(\tilde{n}_{\boldsymbol{\kappa}})$: *H*-theorem $d\mathcal{S}/dz \geqslant 0$



Rayleigh-Jeans:

$$\boxed{\tilde{n}_\kappa^{eq} = \frac{T}{\tilde{\beta}_\kappa - \mu}}$$

→ No heat bath: (T, μ) Lagrange multipliers related to the conservation (*E*, *N*)

→ Because of finite number of modes : (*N*, *E*) do not diverge at RJ equilibrium

Aschieri, Garnier, Michel, Doya, Picozzi -- PRA (2011)

Reminder: Wave condensation in a trap *V(r)*

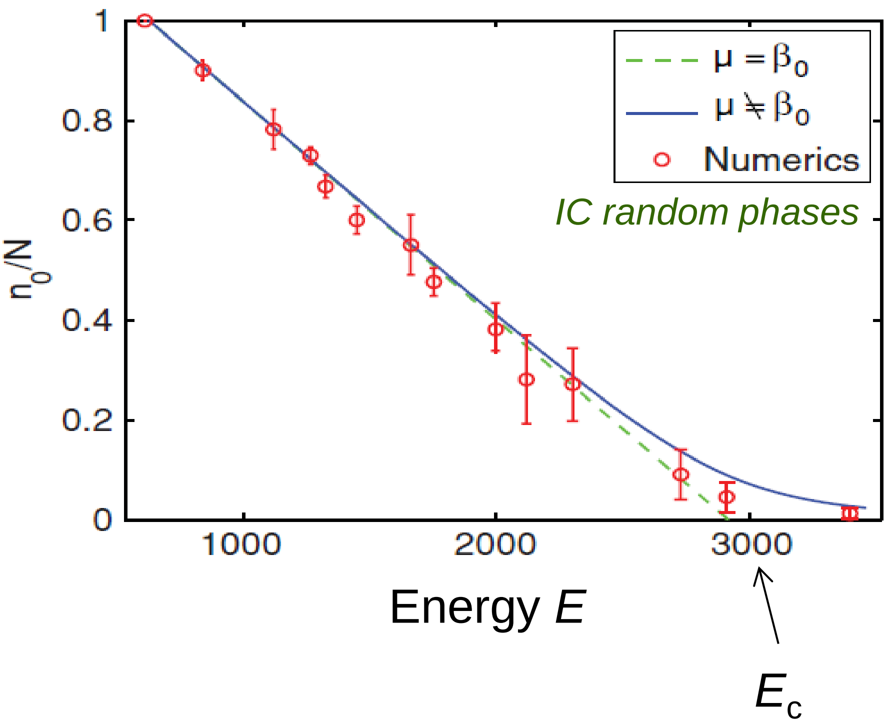
Parabolic trap: Wave condensation takes place in the thermodynamic limit in 2D

$$i\partial_z \psi = -\alpha \nabla^2 \psi + V(\mathbf{r})\psi + \gamma |\psi|^2 \psi$$

Critical energy to condensation properly defined
→ Only depends on the geometry of the waveguide

$$\boxed{E_c = E_0 + N V_0/2 = \frac{N V_0}{2}\left(1 + \frac{2\beta_0}{V_0}\right)}$$

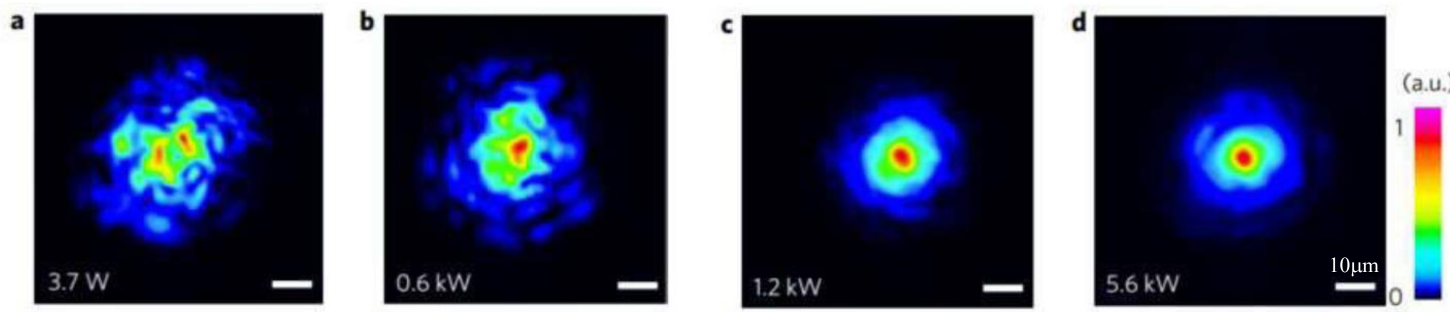
$$\begin{cases} \dfrac{n_0}{N}(\mu) = \dfrac{1}{-\mu \sum_m \dfrac{1}{\beta_m - \beta_0 - \mu}} \\ \langle H \rangle(\mu) = N \dfrac{\sum_m \dfrac{\beta_m}{\beta_m - \beta_0 - \mu}}{\sum_m \dfrac{1}{\beta_m - \beta_0 - \mu}} + \langle U \rangle(\mu) \end{cases}$$



→We have a description of light condensation in a multimode fiber
→ However…..!

Aschieri, Garnier, Michel, Doya, Picozzi -- PRA (2011)

Spatial beam self-cleaning in a multimode optical fiber



Limoges – Dijon - Brescia consortium

Krupa et al., Nature Photon. 11, 237 (2017)

Cornell Un.

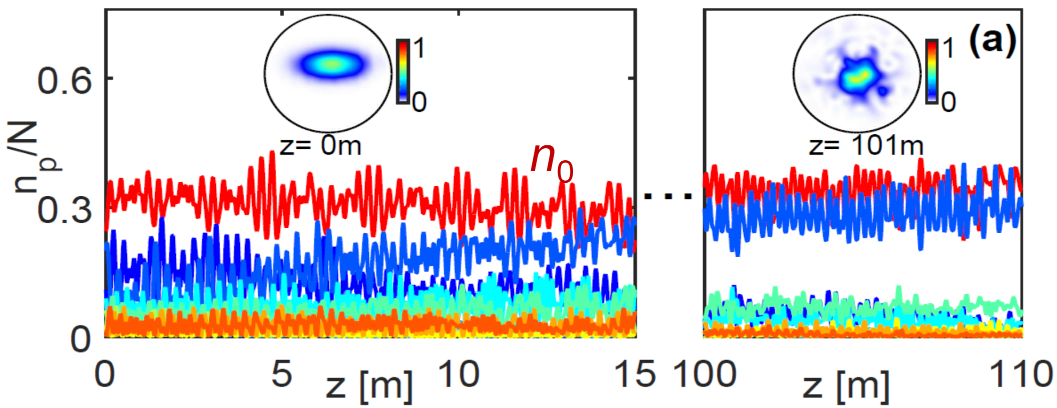
Wright et al., Nature Photon. 10, 771 (2016)

→ *fiber length* ~ 12m

The theory of light condensation in MMFs does NOT explain these experiments:

→ ***Propagation lengths required to achieve condensation are too much important***

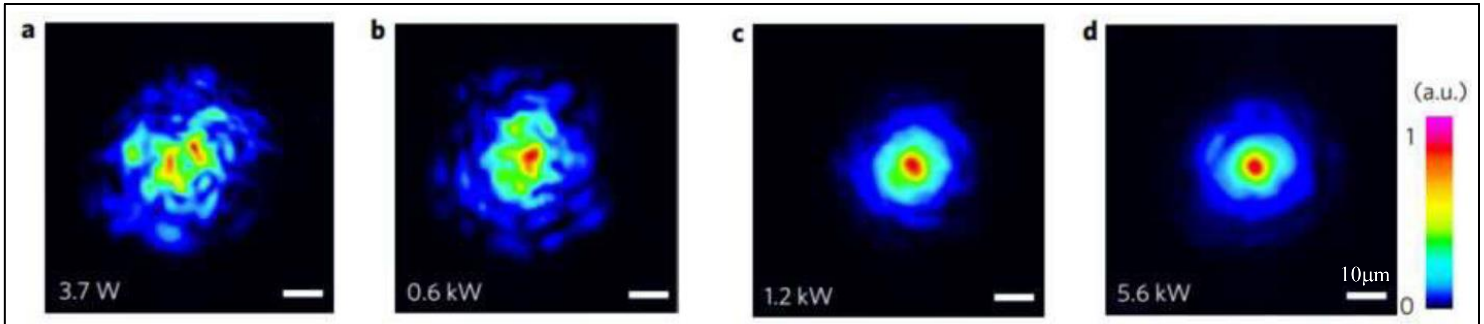
Coherent initial condition:



- → Oscillatory behavior
- → Strong phase-correlation
- → Freezes the thermalization process
- → No Condensation with the short propagation length available in the experiments

→ Need mechanism that breaks the strong the phase-correlation

Spatial beam self-cleaning in a multimode optical fiber



Limoges – Dijon - Brescia collaboration

Krupa et al., Nature Photon. 11, 237 (2017)

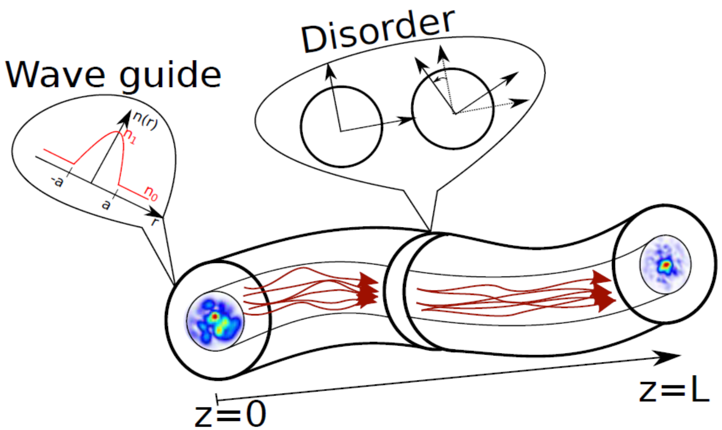
Cornell Un.

Wright et al., Nature Photon. 10, 771 (2016)

To understand these experiments: Important ingredient:

→ **Impact of disorder** : bending and stretching of the fiber introduce linear random coupling among the modes (polar. random fluctuations)

→ Disorder breaks the coherent phase dynamics among the modes



Interplay Disorder & Nonlinearity:

→ vast subject that covers many research fields

→ *Thermalization vs Anderson localization ?*

Wang, Fu, Zhang, Zhao, PRL 124, 186401 (2020)

Nazarenko, Soffer, Tran, Entropy 21, 823 (2019)

→ Here: disorder depends on (‘time’) z variable

No Anderson localization

→ Main result: Disorder accelerates the dynamics of thermalization

Wave turbulence kinetic equation accounting for *weak* disorder

Weak disorder is the leading order contribution that originates in polarization random fluctuations

- Key point: Conservative disorder introduces an effective dissipation (Furutsu-Novikov theorem)
  → modifies the regularization of wave resonances

→ modal NLS eqn : $i\partial_z \boldsymbol{A}_p = \beta_p \boldsymbol{A}_p + \mathbf{W}_p(z)\boldsymbol{A}_p - \gamma \boldsymbol{F}_p(\boldsymbol{A})$

$L_{disor} = (\Delta\beta)^{-1} \ll L_{kin}$

$$\begin{cases} \mathbf{W}_p(z) = \sum_{j=0}^{3} \nu_{p,j}(z)\boldsymbol{\sigma}_j \\ \langle \nu_{p,j}(z)\nu_{p',j'}(z')\rangle = \sigma_\beta^2 \delta^K_{pp'}\delta^K_{jj'}\mathcal{R}\left(\dfrac{z-z'}{l_\beta}\right) \\ \Delta\beta = \sigma_\beta^2 l_\beta \end{cases}$$

$\downarrow\; n_p(z) = \langle |\boldsymbol{A}_p(z)|^2\rangle$

→

$$\begin{cases} \partial_z n_p(z) = \dfrac{2\gamma^2}{9\Delta\beta}\displaystyle\sum_{q,l,m} \delta^K_{\beta_q+\beta_l-\beta_m-\beta_p}|S_{pqlm}|^2 M(\boldsymbol{n}) + \dfrac{16\gamma^2}{27\Delta\beta}\sum_q \delta^K_{\beta_q-\beta_p}|s_{pq}(\boldsymbol{n})|^2(n_q-n_p) \\ M(\boldsymbol{n}) = n_q n_l n_p + n_q n_l n_m - n_m n_p n_l - n_m n_p n_q \\ s_{pq}(\boldsymbol{n}) = \sum_{m'} S_{pqm'm'} n_{m'} \end{cases}$$

$\Delta\beta$: effective strength of disorder β_0 : fundamental eigenvalue

→

Conserves: $N = \sum_p n_p(z)$

$E = \sum_p \beta_p n_p(z)$

H-theorem: $\mathcal{S}(z) = \sum_p \log\left(n_p(z)\right)$ → $n_p^{eq} = T/(\beta_p - \mu)$

Wave turbulence kinetic equation accounting for disorder

$$i\partial_z \boldsymbol{A}_p = \beta_p \boldsymbol{A}_p + \mathbf{W}_p(z)\boldsymbol{A}_p - \gamma \boldsymbol{F}_p(\boldsymbol{A})$$: Kerr nonlinearity $\qquad L_{disor} = (\Delta\beta)^{-1} \ll L_{kin}$

$$n_p(z) = \langle |\boldsymbol{A}_p(z)|^2 \rangle$$

$$\partial_z n_p(z) = \frac{2\gamma^2}{9\Delta\beta} \sum_{q,l,m} \delta^K_{\beta_q+\beta_l-\beta_m-\beta_p} |S_{pqlm}|^2 M(\boldsymbol{n}) + \frac{16\gamma^2}{27\Delta\beta} \sum_q \delta^K_{\beta_q-\beta_p} |s_{pq}(\boldsymbol{n})|^2 (n_q - n_p)$$

$$M(\boldsymbol{n}) = n_q n_l n_p + n_q n_l n_m - n_m n_p n_l - n_m n_p n_q$$

$$s_{pq}(\boldsymbol{n}) = \sum_{m'} S_{pqm'm'} n_{m'}$$

$\Delta\beta$: effective strength of disorder $\qquad \beta_0$: fundamental eigenvalue

Conserves: $N = \sum_p n_p(z)$

$E = \sum_p \beta_p n_p(z)$

H-theorem: $\mathcal{S}(z) = \sum_p \log\left(n_p(z)\right) \rightarrow n_p^{eq} = T/(\beta_p - \mu)$

The characteristic lengths (`times') of thermalization in the presence and the absence of disorder scale as:

$$\boxed{\zeta_{th}^{disor}/\zeta_{th}^{ord} \sim \Delta\beta/\beta_0 \sim 10^{-3} \ll 1}$$

typical experimental parameters

Significant acceleration of thermalization mediated by disorder

Disorder modifies the regularization of wave resonances

- Derived WT kinetic equation accounting for a structural disorder of the nl medium
  → dominant contribution of polarization fluctuations (Agrawal's model: Mumtaz et al JLT 2013)
- Key point: **Conservative disorder introduces an effective dissipation** (Furutsu-Novikov theorem)
  → modifies the regularization of wave resonances

- Equation for 4th order moment is governed by an effective forced-damped oscillator eqn

4th moment disorder 6th moment

$$\partial_z J_{qlmp} = (-6\Delta\beta + i\Delta\omega_{qlmp}) J_{qlmp} + i\gamma \langle Y_{qlmp} \rangle$$

$$\Delta\omega_{qlmp} = \beta_q + \beta_l - \beta_m - \beta_p$$ frequency resonance (β_p mode eigenvalue)

$$L_{lin} = \beta_0^{-1} \ll L_{disor} = (\Delta\beta)^{-1} \ll L_{nl}$$

- Discrete wave turbulence

$\beta_0 \simeq 5 \times 10^3 \mathrm{m}^{-1} \rightarrow \min(|\Delta\omega_{qlmp}|) = \beta_0 \gg 1/L_{nl}$ only exact resonances can contribute

a) $\Delta\omega_{qlmp} = 0 : \; L_{disor} = (\Delta\beta)^{-1} \ll L_{nl}$

$G(z) = H(z) \exp(i\Delta\omega_{qlmp} z - 6\Delta\beta z)$ decays on a length << L_{nl}

$$J_{qlmp}(z) \simeq \frac{i\gamma}{6\Delta\beta} \langle Y_{qlmp} \rangle (z)$$

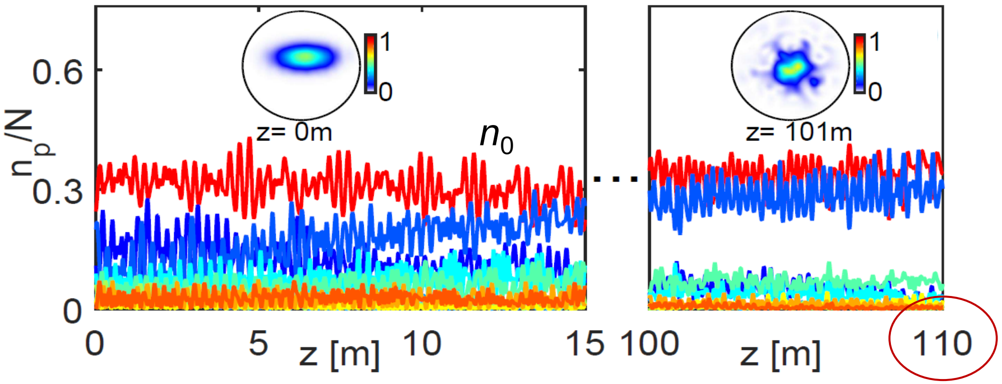
b) $\Delta\omega_{qlmp} \neq 0 : \; L_{lin} = \beta_0^{-1} \ll L_{nl}$ → rapid oscillating phase
→ vanishing contribution of quasi-resonances

$$\boxed{J_{qlmp}(z) \simeq \frac{i\gamma}{6\Delta\beta} \langle Y_{qlmp} \rangle (z) \delta^K(\Delta\omega_{qlmp})}$$

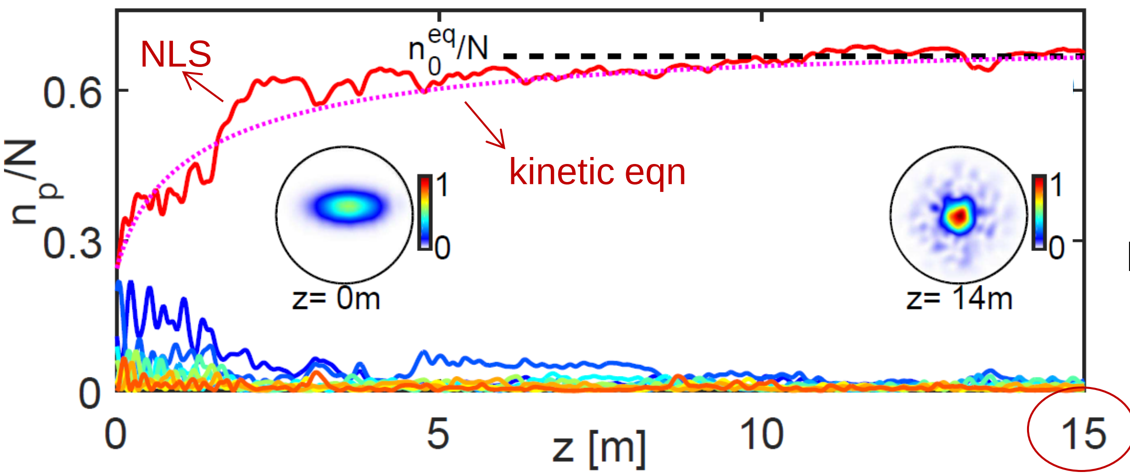
→ **Singularity of a wave resonance is regularized by the dissipation due to disorder**

Numerical simulations vs Theory

Without disorder: → Coherent regime of modal interaction → no condensation



With disorder: → Breaks the coherent regime → Fast Thermalization & Condensation



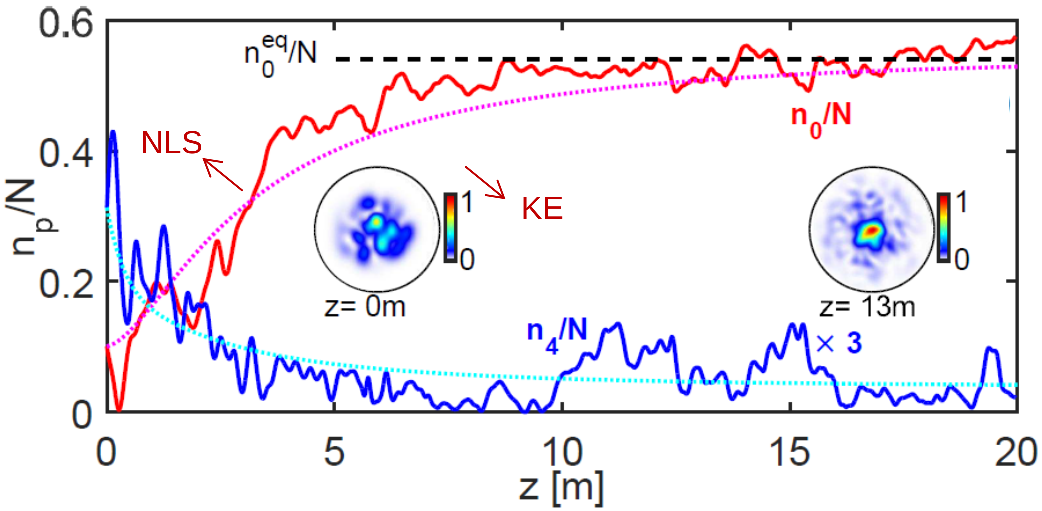
------- Kinetic eqn

——— NLS eqn

No adjustable parameters

$$\partial_z n_p(z) = \frac{2\gamma^2}{9\Delta\beta} \sum_{q,l,m} \delta^K_{\beta_q+\beta_l-\beta_m-\beta_p} |S_{pqlm}|^2 M(\boldsymbol{n}) + \frac{16\gamma^2}{27\Delta\beta} \sum_q \delta^K_{\beta_q-\beta_p} |s_{pq}(\boldsymbol{n})|^2 (n_q - n_p)$$

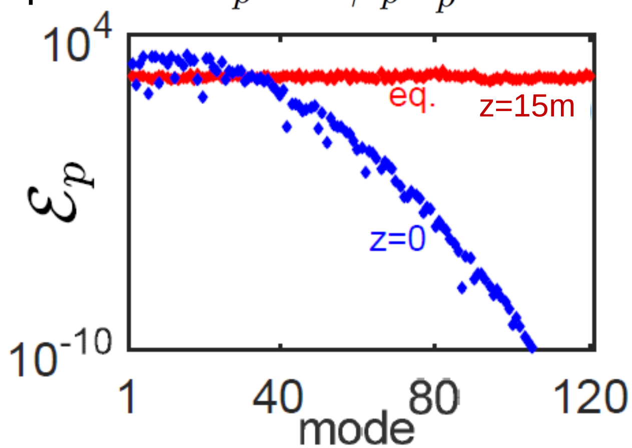
Phase transition to condensation



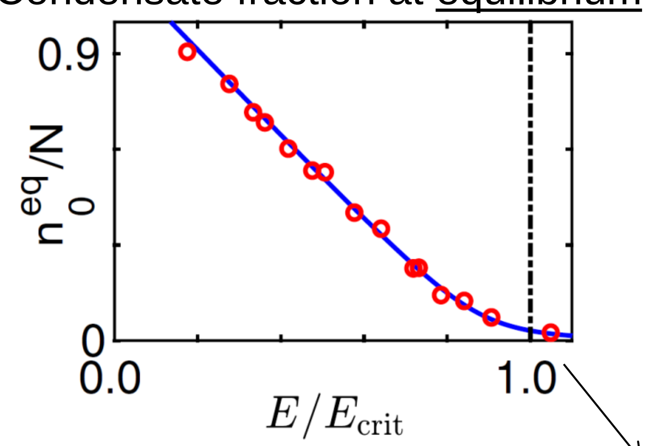
No adjustable parameters

Thermalization is characterized by

→ Energy equipartition: $\mathcal{E}_p = \beta_p n_p^{eq} \simeq T$



Condensate fraction at <u>equilibrium</u>



phase transition

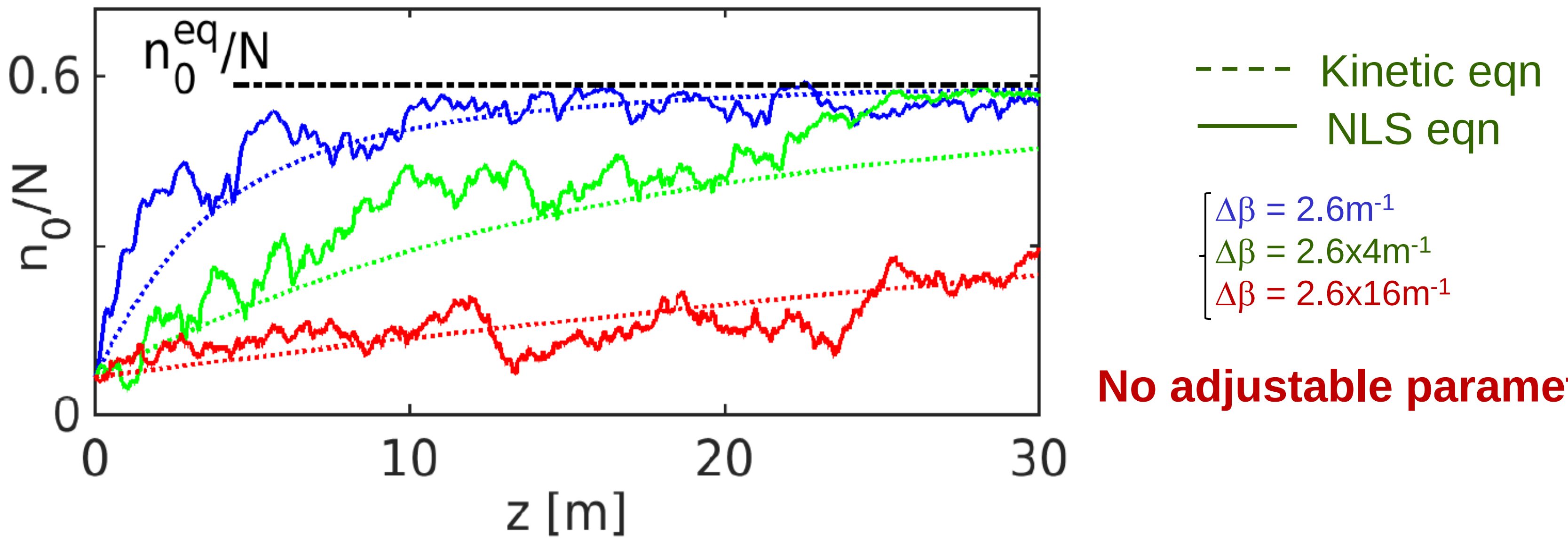
$$E_{\mathrm{crit}} = \frac{NV_0}{2}\left(1 + \frac{2\beta_0}{V_0}\right)$$

→ Energy flows to high-order modes (direct cascade)

→ Power flows to the fudamental mode (inverse cascade)

Scaling of acceleration of thermalization

→ by decreasing disorder $\Delta\beta$



No adjustable parameters

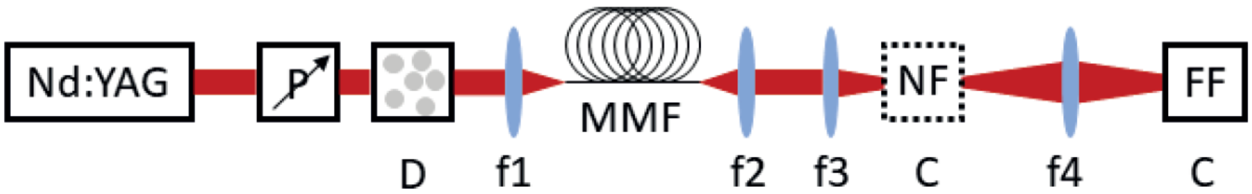
The lower the magnitude of disorder → the faster the thermalization

Because the singularity of a wave resonance is regularized by the dissipation due to disorder

$$\partial_z n_p(z) = \frac{2\gamma^2}{9\Delta\beta} \sum_{q,l,m} \delta^K_{\beta_q+\beta_l-\beta_m-\beta_p} |S_{pqlm}|^2 M(\boldsymbol{n}) + \frac{16\gamma^2}{27\Delta\beta} \sum_q \delta^K_{\beta_q-\beta_p} |s_{pq}(\boldsymbol{n})|^2 (n_q - n_p)$$

Experimental observation of the transition to condensation

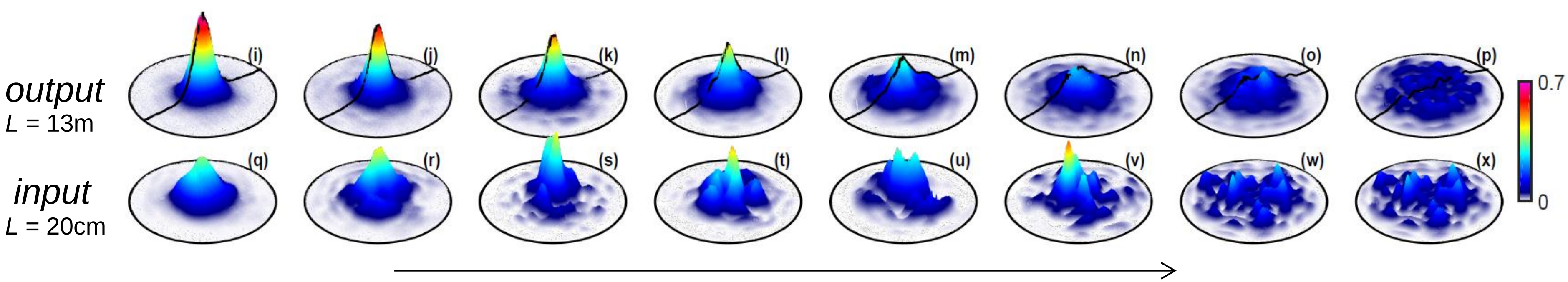
Experiment realized in Dijon by K. Baudin, A. Fusaro, K. Krupa, G. Millot



→ diffuser: vary the energy $E = \sum_p \beta_p |\boldsymbol{a}_p|^2$ while keeping constant $N = \sum_p |\boldsymbol{a}_p|^2$ = *const*

→ weakly nonlinear regime: $L_{lin} \simeq \beta_0^{-1} \sim 0.2\mathrm{mm} \ll L_{nl} \sim 0.5\mathrm{m}$ (No coherent structures)

→ verified conservation of (E, N) during the propagation through the fiber



energy *E ('temperature')*

fiber length L =13m, fiber radius R = 26µm, 2x120 modes, N =7kW

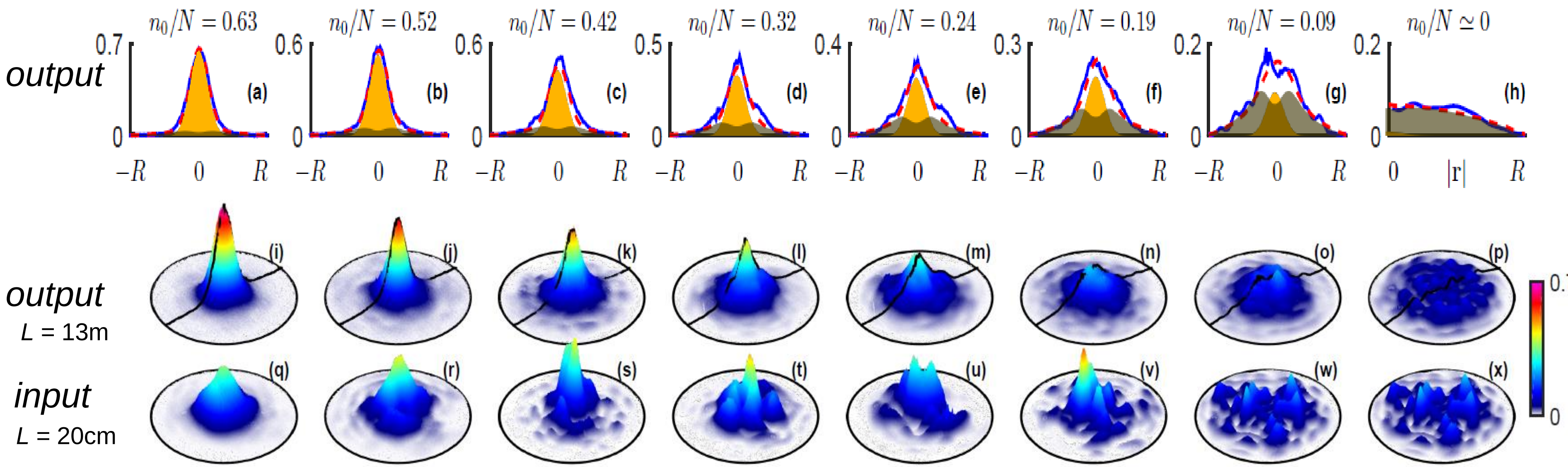
Preliminary comparison with Rayleigh-Jeans equilibrium (single-shot)

Experimental measurements

Condensate contribution (fundamental mode) $I_{condens}(\boldsymbol{r}) = n_0 u_0^2(\boldsymbol{r})$

Incoherent contribution (all modes p ≠ 0) $I_{incoh}(r) = \sum_{p\neq 0} n_p^{eq} u_p^2(r)$

sum of condensate and incoherent contributions



energy E

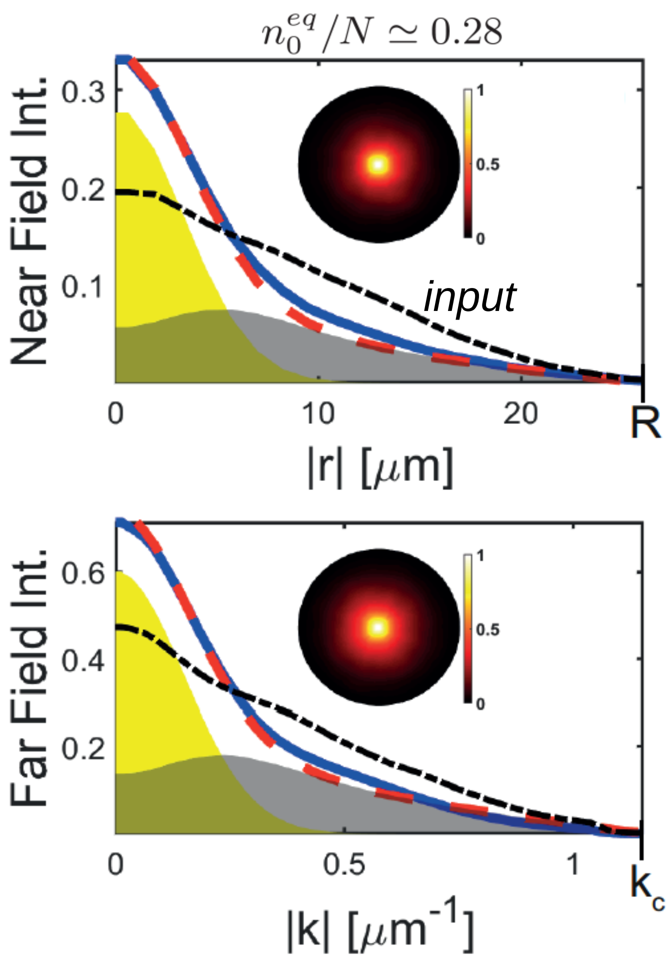
$N = \sum_p |\boldsymbol{A}_p|^2(z)$ = const

Preliminary study: compare single-shot measurements (i.e. speckle beam) with RJ equilibrium

→ But RJ eq. is a statistical distribution → comparison requires an average over the realizations

Quantitative comparison with RJ theory:

→ Average over the experimental realizations for a given pair (N, E)



(N,E) measured experimentally

$$\begin{cases} N = \sum_p n_p^{eq} \\ E = \sum_p \beta_p n_p^{eq} \\ n_p^{eq} = T/(\beta_p - \mu) \end{cases}$$

→ *determine the unique pair (μ,T)*

RJ intensity: $I^{eq}(r) = \sum_p n_p^{eq} u_p^2(\boldsymbol{r})$

$u_p(\boldsymbol{r})$: Hermite-Gauss modes fixed by the MMF in the exp.

→ ***No adjustable parameters***

Experimental measurements

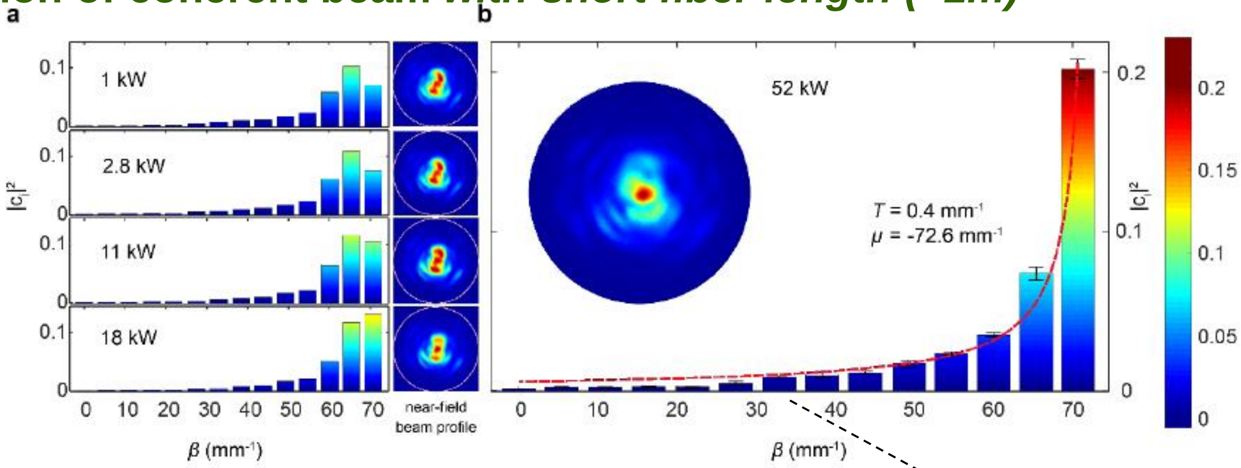
Condensate contribution (fundamental mode) $I_{condens}(\boldsymbol{r}) = n_0 u_0^2(\boldsymbol{r})$

Incoherent contribution (all modes p ≠ 0) $I_{incoh}(r) = \sum_{p \neq 0} n_p^{eq} u_p^2(r)$

sum of condensate and incoherent contributions

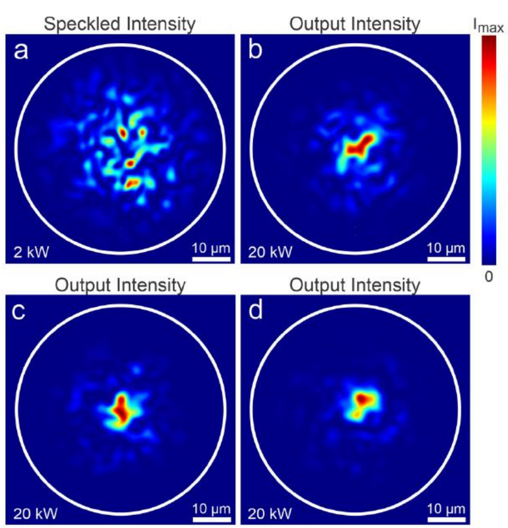
Direct observations of thermalization to a Rayleigh–Jeans distribution in multimode optical fibres
Pourbeyram, Sidorenko, Wu, Bender, Wright, Christodoulides, and Wise
Nature Physics – April 2022

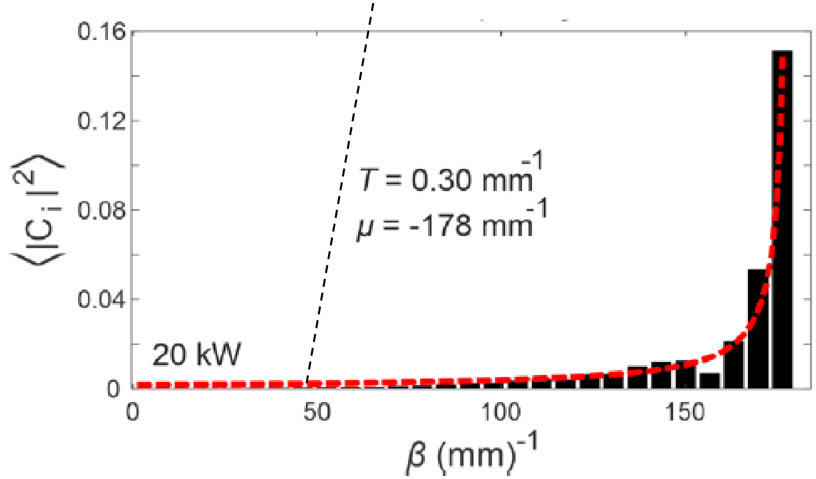
Injection of coherent beam *with short fiber length (~1m)*



Injection of speckle beam *with short fiber length (~1m)*

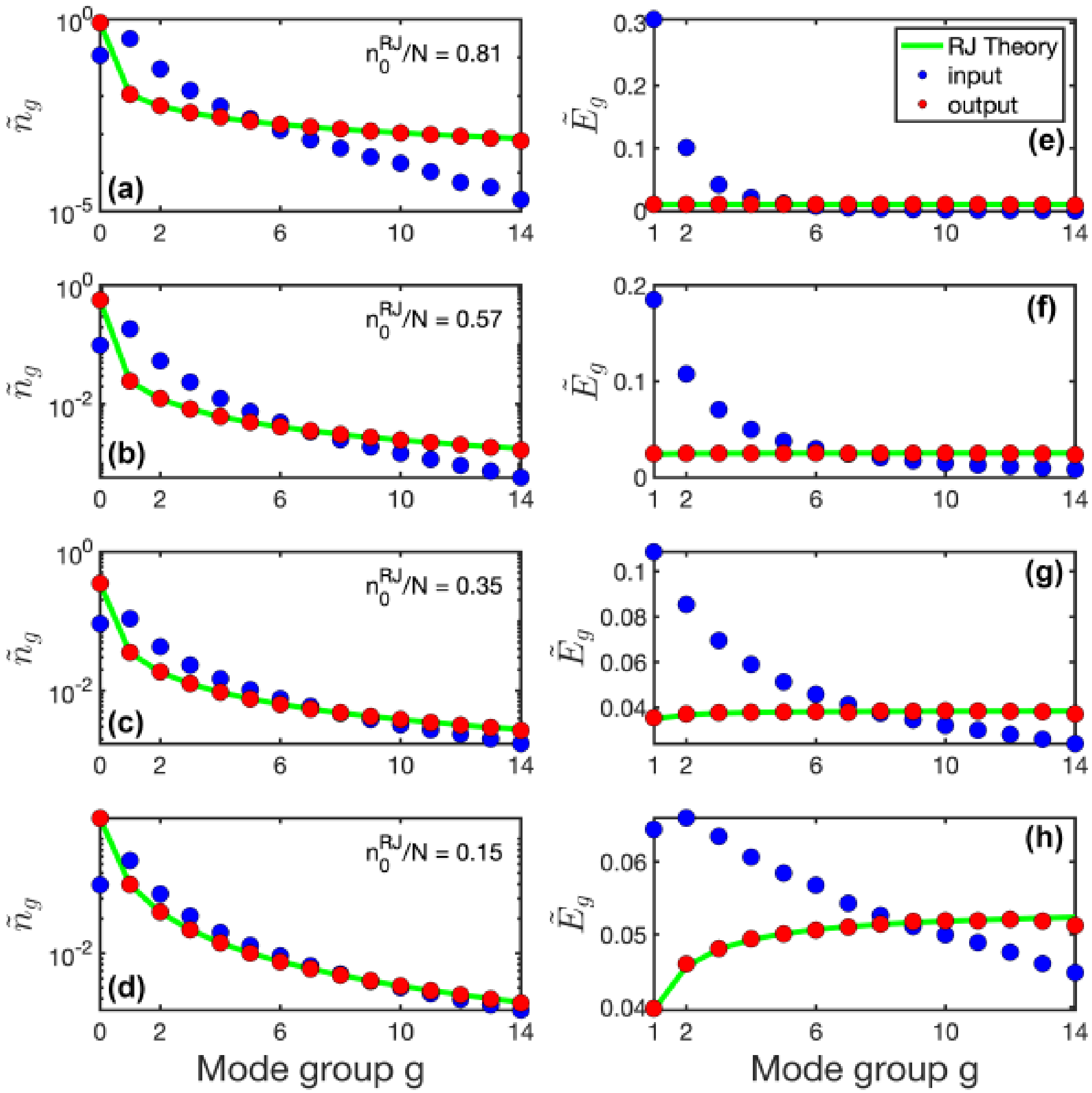
discrepancy for higher order modes





→ **For *short fiber length the impact of disorder is severely limited !***

More precise comparison with a modal decomposition
→ No adjustable parameters (!)



- Injection of speckle beams with an appropriate averaging within small energy intervals
- Long fiber length (12m) → enhanced impact of disorder

Gerchberg-Saxton algorithm
→ allows to retrieve the phase profile from NF & FF intensity distributions measured experimentally

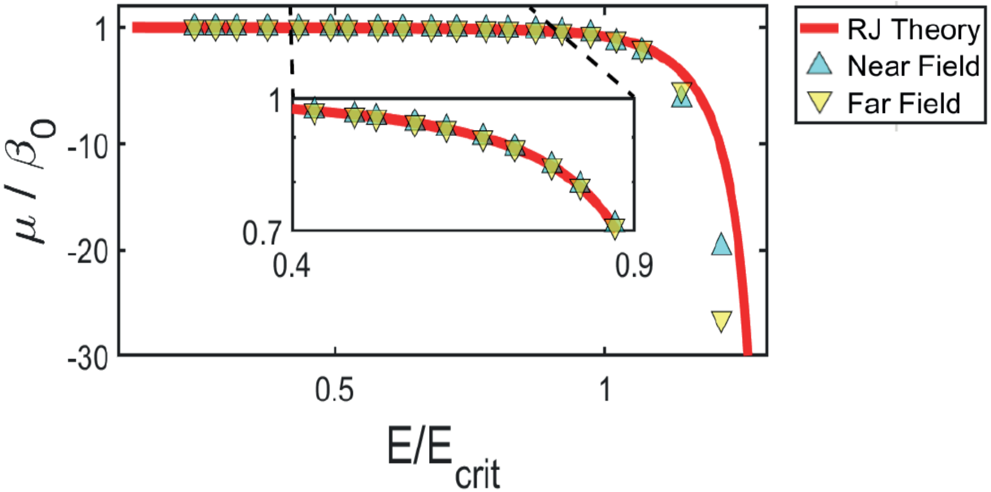
$$\begin{cases} g = 0, .., g_{max} - 1 \\ M = g_{max}(g_{max}+1)/2 = 120 \text{ modes} \end{cases}$$

Indexes the mode group: 15 groups of degenerate modes

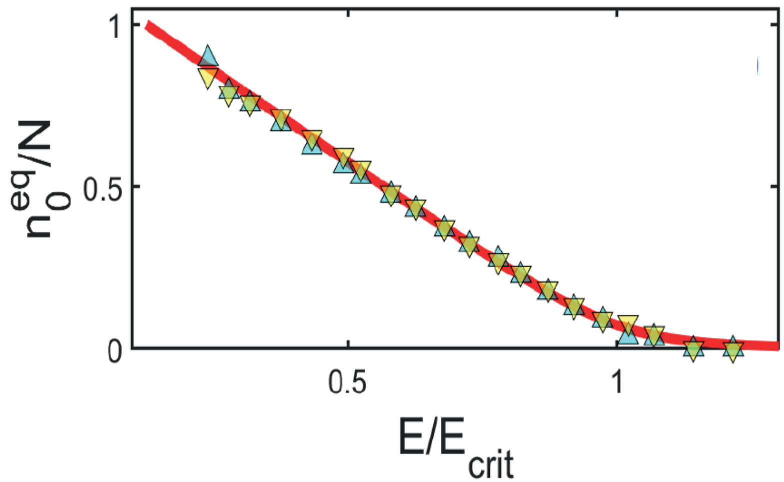
$$\begin{cases} E_p = (\beta_p - \beta_0) n_p \\ \tilde{E}_g = \beta_0 g \tilde{n}_g \end{cases}$$

Observation of the transition to Rayleigh-Jeans condensation

chemical potential



condensate fraction



By decreasing the energy E : $\mu \to \beta_0^-$

→ divergence RJ eq. $n_p^{eq} = T/(\beta_p - \mu)$

→ Macroscopic populat. fundamental mode

→ Phase transition to condensation

<u>Theory</u>: no adjustable parameters (β_p, M = 120) fixed by the fiber used in the experiment

<u>RJ theory:</u>

$$\left\{ \begin{aligned} \frac{n_0^{eq}}{N}(\mu) &= \frac{1}{-(\mu - \beta_0)\sum_p (\beta_p - \mu)^{-1}}, \\ \frac{E}{E_{\rm crit}}(\mu) &= \frac{\sum_p \frac{\beta_p}{\beta_p - \mu}}{(1 + (M-1)/\varrho)\sum_p \frac{\beta_0}{\beta_p - \mu}}, \\ \beta_p &= \beta_0 (p_x + p_y + 1) \end{aligned} \right.$$

fiber length L =13m, fiber radius R = 26μm, 2x120 modes, P =7kW

Thermodynamics of classical condensation: Specific heat

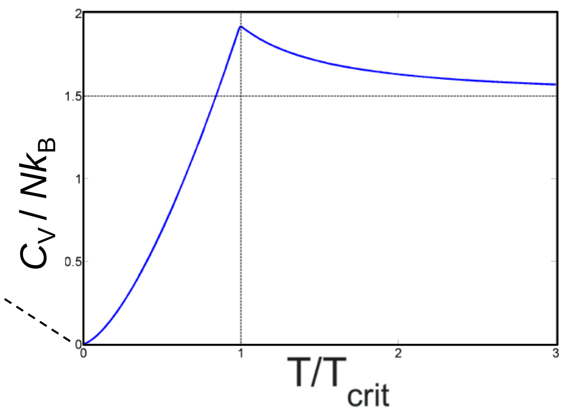
Wu, Hassan, Christodoulides, Nature Photonics 13, 776 (2019)
Thermodynamic theory of highly multimoded nonlinear optical systems

→ Behavior of the specific heat across the transition to condensation

- Quantum Bose-Einstein transition:

$$C_V \to 0$$

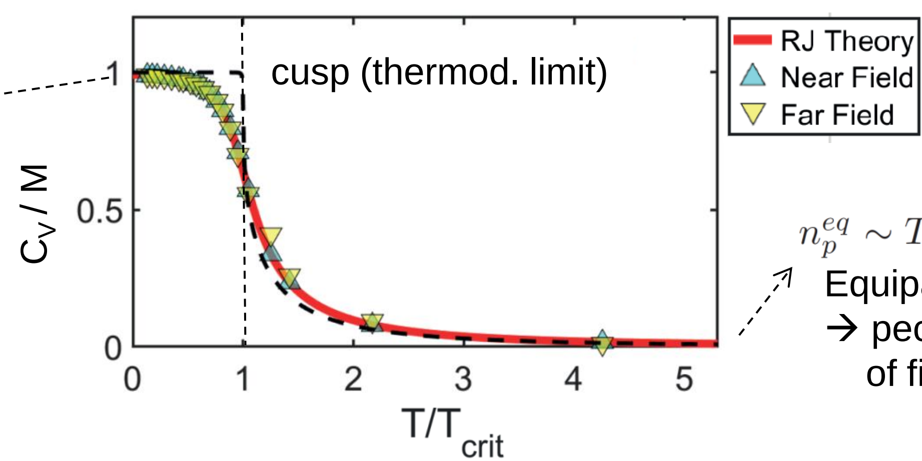
quantum notion of frozen degrees of freedom (3rd law thermodynamics)



Bose gas behaves as a classical gas: Energy equipartition $C_V(T)$ =const

- Classical condensation of waves: $C_V = (\partial E/\partial T)_{N,M}$ $\quad C_V(E) = M - \frac{\{\sum_p [\beta_p - \mu(E)]^{-1}\}^2}{\sum_p [\beta_p - \mu(E)]^{-2}}$

Equipartition of ***energy***:
$C_V(T)$ =const



$$C_V \to 0$$
$$n_p^{eq} \sim T/(-\mu) \text{ for } -\mu \gg \beta_p$$

Equipartition of ***power***
→ peculiar feature of finite nb modes

Conclusion

- Derived a WT kinetic eqn accounting for the presence of weak disorder (weak random mode coupling))
  → Rate of Thermalization & Condensation is significantly increased by weak disorder

- Turbulence in MMFs → ***Discrete turbulence***
  → parabolic MMFs : Lot of exact resonances
  → Non-parabolic (step-index) MMFs: Only poorly efficient quasi-resonances
  → Freezing of thermalization & condensation → Explain why No beam cleaning in step-index MMFs

- Reported experimental observation of RJ condensation of classical light
  → Quantitative agreement between exp. results & RJ equilibrium theory

- Derivation of a kinetic eqn in the presence of strong disorder → interplay disorder & nonlinearity
  → quantitative agreement with simulations

- Perspective: Finite number of modes → Existence of upper bound for the energy (E_{max})
  → Entails the existence of Negative Temperature equiibrium states : $T < 0$

→ Special thanks to the students K. Baudin, A. Fusaro, N. Berti

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EPL134, 14001 (2021)