A priori and a posteriori error analysis in $\boldsymbol{H}(\mathbf{curl})$: localization, minimal regularity, and p-optimality

Martin Vohralík*
joint work with Théophile Chaumont-Frelet†

*Inria Paris and CERMICS
†Université Côte d'Azur, Inria, CNRS, LJAD

We design a stable local commuting projector from the entire infinite-dimensional Sobolev space $\boldsymbol{H}(\mathbf{curl})$ onto its finite-dimensional subspace formed by the Nédélec piecewise polynomials on a tetrahedral mesh. The projector is defined by simple piecewise polynomial projections and is stable in the $\boldsymbol{L}^2$ norm, up to data oscillation. It in particular allows to establish the equivalence of local-best and global-best approximations in $\boldsymbol{H}(\mathbf{curl})$. This in turn yields to a priori error estimates under minimal Sobolev regularity in $\boldsymbol{H}(\mathbf{curl})$, localized elementwise, optimal both in the mesh size h and in the polynomial degree p. In the heart of the projector, there is an $\boldsymbol{H}(\mathbf{curl})$-conforming flux reconstruction procedure. This itself leads to guaranteed, fully computable, constant-free, and p-robust a posteriori error estimates in $\boldsymbol{H}(\mathbf{curl})$. Details can be found in [1–3].

[1] Chaumont-Frelet, Théophile and Vohralík, Martin. Equivalence of local-best and global-best approximations in $\boldsymbol{H}(\mathbf{curl})$. Calcolo 58 (2021), 53.

[2] Chaumont-Frelet, Théophile and Vohralík, Martin. p-robust equilibrated flux reconstruction in $\boldsymbol{H}(\mathbf{curl})$. based on local minimizations. Application to a posteriori analysis of the curlcurl problem. HAL preprint 03227570, submitted for publication, 2022.

[3] Chaumont-Frelet, Théophile and Vohralík, Martin. A stable local commuting projector and optimal hp approximation estimates in $\boldsymbol{H}(\mathbf{curl})$. In preparation, 2022.