Weighted analytic regularity for the integral fractional Laplacian in polygons and application to *hp*-FEM

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We study the Dirichlet problem for the integral fractional Laplacian in a polygon Ω with analytic right-hand side. We show the solution to be in a weighted analyticity class that captures both the analyticity of the solution in Ω and the singular behavior near the boundary. Near the boundary the solution has an anisotropic behavior: near edges but away from the corners, the solution is smooth in tangential direction and higher order derivatives in normal direction are singular at edges. At the corners, also higher order tangential derivatives are singular. This behavior is captured in terms of weights that are products of powers of the distances from edges and corners.

The proof of the regularity assertions is based on the Caffarelli-Silvestre extension, which realizes the non-local fractional Laplacian as a Dirichlet-to-Neumann map of a (degenerate) elliptic boundary value problem.

We employ our analytic regularity to show exponential convergence of high order finite element methods (*hp*-FEM) on meshes that are geometrically refined towards both edges and corners. The geometric refinement towards edges results in anisotropic meshes away from corners. The use of such anisotropic elements is crucial for the exponential convergence result.

[1] M. Faustmann, C. Marcati, J. M. Melenk and, C. Schwab. Weighted analytic regularity for the integral fractional Laplacian in polygons. arXiv:2112.08151