Sharp bounds on Helmholtz impedance-to-impedance maps and application to overlapping domain decomposition

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This talk describes sharp bounds on certain impedance-to-impedance maps for the Helmholtz equation at high-frequency, with these bounds proved using semiclassical defect measures.

Our main motivation for studying these maps is in the analysis of overlapping domain-decomposition methods for the high-frequency Helmholtz equation. The recent paper [Gong-Gander-Graham-Lafontaine-Spence] studied a parallel overlapping Schwarz method for the Helmholtz equation with impedance boundary conditions on the subdomains. This paper related the error-propagation operator for this DD method to certain impedance-to-impedance maps and they compositions, and thus formulated sufficient conditions for convergence of the DD method in terms of properties of such maps.

For a model decomposition with two subdomains and sufficiently-large overlap, our results show that the parallel Schwarz method is power contractive, independent of the wavenumber. For strip-type decompositions with many subdomains, we show that the composite impedance-to-impedance maps, in general, behave badly with respect to the wavenumber; nevertheless, by proving results about the composite maps applied to a restricted class of data, we give insight into the wavenumber-robustness of the parallel Schwarz method observed in numerical experiments.