Crack propagation based on Griffith's fracture model

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The propagation of cracks in a brittle material typically relies on the Griffith fracture criterion which states that a crack will only advance if the crack driving force (energy release rate) equals to the crack resistance force (fracture toughness κ). Given a two dimensional domain with an already existing crack, the evolution system describing the crack extension is of the following type:

$$0 \in \partial\mathcal{R}(\dot{s}(t)) + \mathrm{D}\mathcal{I}(t, s(t)), \qquad s(0) = s_0\,. \tag{1}$$

Here, $s : [0, T] \to [0, L]$ denotes the crack length at time t; $\mathcal{R}(\dot{s}) = \kappa\dot{s}$ if $\dot{s} \geq 0$ and $\mathcal{R}(\dot{s}) = \infty$ otherwise is a dissipation potential encoding among others the unidirectionality of crack propagation; $\mathcal{I}(t, s)$ is the stored elastic energy for a given crack length s and load state at time t. This system belongs to the class of rate-independent systems.

In general, the mapping $s \mapsto \mathcal{I}(t, s)$ is not convex and hence system (1) might not have a global solution that is continuous in time. In the past 20 years, several weak solution concepts were developed for rate-independent systems with nonconvex energies, [4]. We focus here on a solution concept that is based on a vanishing viscosity argument leading to *balanced viscosity solutions*. In the lecture, we give a short introduction to the evolution model and solution concepts. We then focus on the numerical approximation of such solutions by fully discretized schemes (time and space): We present convergence theorems and illustrate their behavior for a simple example, [2,3]. The convergence proofs heavily rely on the spatial regularity properties of the solutions and in particular on the singular behavior of the solutions in the vicinity of the crack tip, [1].

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