Crack propagation based on Griffith’s fracture model
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The propagation of cracks in a brittle material typically relies on the Griffith fracture criterion which states that a crack will only advance if the crack driving force (energy release rate) equals to the crack resistance force (fracture toughness $\kappa$). Given a two-dimensional domain with an already existing crack, the evolution system describing the crack extension is of the following type:

$$0 \in \partial R(\dot{s}(t)) + D\mathcal{I}(t, s(t)), \quad s(0) = s_0.$$

Here, $s : [0, T] \to [0, L]$ denotes the crack length at time $t$; $R(\dot{s}) = \kappa \dot{s}$ if $\dot{s} \geq 0$ and $R(\dot{s}) = \infty$ otherwise is a dissipation potential encoding among others the unidirectionality of crack propagation; $\mathcal{I}(t, s)$ is the stored elastic energy for a given crack length $s$ and load state at time $t$. This system belongs to the class of rate-independent systems.

In general, the mapping $s \mapsto \mathcal{I}(t, s)$ is not convex and hence system (1) might not have a global solution that is continuous in time. In the past 20 years, several weak solution concepts were developed for rate-independent systems with nonconvex energies, [4]. We focus here on a solution concept that is based on a vanishing viscosity argument leading to balanced viscosity solutions. In the lecture, we give a short introduction to the evolution model and solution concepts. We then focus on the numerical approximation of such solutions by fully discretized schemes (time and space): We present convergence theorems and illustrate their behavior for a simple example, [2,3]. The convergence proofs heavily rely on the spatial regularity properties of the solutions and in particular on the singular behavior of the solutions in the vicinity of the crack tip, [1].


