Geometric singularities and high-order finite elements for the integral fractional Laplacian

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Diffusion processes beyond Brownian motion have recently attracted significant interest from different communities in mathematics, the physical and biological sciences. They are described by partial differential equations involving nonlocal operators with singular non-integrable kernels, such as fractional Laplacians.

We consider the sharp boundary regularity of solutions to the Dirichlet problem for the integral fractional Laplacian on a bounded polygonal domain in $\mathbb{R}^2$. It is defined via the extension method, as a Dirichlet-to-Neumann operator for a degenerate elliptic problem in a half space in 3 dimensions. We discuss techniques from geometric microlocal analysis to analyse the regularity of solutions, with particular emphasis on asymptotic expansions at boundaries and corners. Applications to the a priori analysis of $h$, $p$ and $hp$-versions of the finite element method are presented. Numerical experiments illustrate the theoretical results.