A pressure-robust discretization of the Stokes problem on anisotropic meshes

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joint work with Volker Kempf*

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Anisotropic finite element meshes are particularly efficient when the solution of the problem has anisotropic features like edge singularities or boundary layers. Pressure-robustness means that the velocity discretization error does not depend on the pressure approximation. The Crouzeix-Raviart method is known to be inf-sup stable on arbitrary anisotropic meshes, yet not pressure-robust.

Inf-sup stable finite element schemes with discontinuous pressure can be made pressure-robust by a modified discretization of the exterior forcing term using $\boldsymbol{H}(\mathrm{div})$-conforming reconstruction operators like the Raviart-Thomas or Brezzi-Douglas-Marini interpolants, [1].

In order to show that the reconstruction approach works for anisotropic discretizations, it was necessary to investigate the interpolation error for the Raviart-Thomas or Brezzi-Douglas-Marini interpolants on anisotropic elements disclosing subtleties in the definition of the spaces and in shape assumptions, [2,3].

In collaboration with Alexander Linke and Christian Merdon we have generalized the modified Crouzeix-Raviart method to a large class of anisotropic triangulations. Numerical examples confirm the theoretical results, [4,5].

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